First Exact Solutions for Flows of Rate Type Fluids in a Circular Duct that Applies a Constant Couple to the Fluid

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Rotational flow of an Oldroyd-B fluid induced by an infinite circular cylinder that applies a constant couple to the fluid is studied by means of integral transforms. Such a problem is not solved in the existing literature for rate type fluids and the present solutions are based on a simple but important remark regarding the governing equation for the non-trivial shear stress. The solutions that have been obtained satisfy all imposed initial and boundary conditions and can easy be reduced to the similar solutions corresponding to Maxwell, second-grade, and Newtonian fluids performing the same motion. Finally, the influence of material parameters on the velocity and shear stress distributions is graphically underlined.

Key words: Oldroyd-B Fluid; Circular Duct; Constant Couple; Exact Solutions.

1. Introduction

In the last time many motion problems concerning non-Newtonian fluids have been studied by different researchers. Their solutions play an important role in technological applications such as industrial materials and biological fluids. Among the many models that have been used to describe the behaviour of non-Newtonian fluids, the Oldroyd-B model seems to be amenable to analysis and more importantly experimental. This model, that can describe stress-relaxation, creep, and normal stress differences, can be viewed as one of the most successful models for describing the response of many dilute polymeric liquids. It contains as special cases the viscous and Maxwell fluid models.

The motion of a fluid in the neighbourhood of a moving body is of great interest for industry. The flow between cylinders or through a rotating cylinder has applications in the food industry, it being one of the most important and interesting problems of motion near rotating bodies. The velocity distribution for different motions of Newtonian fluids through a circular cylinder is given in [1]. First exact solutions corresponding to motions of non-Newtonian fluids in cylindrical domains seem to be those of Ting [2], Srivastava [3] and Waters and King [4] for second grade, Maxwell, respectively Oldroyd-B fluids. During the time many papers regarding such motions of non-Newtonian fluids have been published. Among them, we here remember only a few of those concerning Oldroyd-B fluids [5 – 12].

However, all above-mentioned papers deal with motion problems in which the velocity is given on the boundary although in some practical problems what is specified is the shear stress [13 – 15], more exactly the force with which the cylinder is moved. To reiterate, in Newtonian mechanics force is the cause and kinematics is the effect (see Rajagopal [16] for a detailed discussion on the same). Moreover, the 'no slip' boundary condition may not be necessary for flows of polymeric fluids that can slide on the boundary. Consequently, the boundary conditions on stresses are meaningful, and Renardy [14] showed how well-posed boundary value problems can be formulated in this way. The first exact solutions for motions of Oldroyd-B fluids when the shear stress is given on a part of the boundary seem to be those of Waters and King [17]. In the last time many solutions for such motions of rate type fluids have been established (see [18 – 22] and the references therein), but all these solutions correspond to differential expressions of the shear stress on the boundary. This is due to their governing equations that, unlike those corresponding to Newtonian and second grade fluids, contain differential expressions acting on the non-trivial shear stresses.

The purpose of this work is to establish exact solutions for the motion of an Oldroyd-B fluid induced by an infinite circular cylinder that applies a constant rotational shear stress to the fluid. These solutions, which can easy be particularized to Maxwell, second-grade, and Newtonian fluids, are obtained using a simple but interesting consequence of the corresponding constitutive and motion equations [23]. They are the first exact solutions of this kind for rate type fluids because the solutions obtained in [18] and [19] do not correspond to a constant shear on the boundary as the authors mentioned there. As a check of results, some known solutions for second grade and Newtonian fluids are recovered as limiting cases of general solutions. Finally, the influence of material parameters on the velocity and shear stress distributions is graphically underlined.

2. Mathematical Formulation of the Problem

The Cauchy stress tensor $\mathbf{T}$ corresponding to an incompressible Oldroyd-B fluid is related to the fluid motion by the relations [5–12]

$$\begin{aligned}&\mathbf{T} = -p\mathbf{I} + \mathbf{S},\\ &\mathbf{S} + \lambda\,(\dot{\mathbf{S}} - \mathbf{LS} - \mathbf{SL}^{\mathrm{T}})\\ &\quad = \mu\left[\mathbf{A} + \lambda_{\mathrm{r}}(\dot{\mathbf{A}} - \mathbf{LA} - \mathbf{AL}^{\mathrm{T}})\right],\end{aligned} \tag{1}$$

where $-p\mathbf{I}$ denotes the indeterminate spherical stress due to the constraint of incompressibility, $\mathbf{S}$ is the extra-stress tensor, $\mathbf{L}$ is the velocity gradient, $\mathbf{A} = \mathbf{L} + \mathbf{L}^{\mathrm{T}}$ is the first Rivlin–Ericksen tensor, μ is the dynamic viscosity, λ and λ_{r} are relaxation and retardation times. The superscript T indicates the transpose operation and the superposed dot denotes the material time derivative. The model characterized by the constitutive equations (1) contains as special cases the upper convected Maxwell model for $\lambda_{\mathrm{r}} = 0$ and the Newtonian fluid model for $\lambda = \lambda_{\mathrm{r}} = 0$. In some special cases like that to be here considered, the governing equations for Oldroyd-B fluids resemble those for second-grade fluids. Consequently, for such flows, the solutions for second-grade fluids as well as those for Maxwell and Newtonian fluids can be obtained as limiting cases of general solutions corresponding to Oldroyd-B fluids.

Let us assume that an incompressible Oldroyd-B fluid is at rest in an infinite circular cylinder of radius R. At time $t = 0^+$ the cylinder begins to turn about its axis due to a constant torque per unit length $2\pi Rf$ [24, Sect. 5]. Owing to the shear, the fluid is gradually moved and we are looking for a velocity field of the form

$$\mathbf{v} = \omega(r,t)\mathbf{e}_\theta\,, \tag{2}$$

where $\mathbf{e}_\theta$ is the unit vector in the θ-direction of the system of cylindrical coordinates r, θ, and z. For such a flow, the constraint of incompressibility is identically satisfied. We also assume that the extra-stress tensor $\mathbf{S}$, as well as the velocity $\mathbf{v}$, is a function of r and t only.

If the fluid is at rest up to the moment $t = 0$, then

$$\mathbf{v}(r,0) = \mathbf{0}\,, \qquad \mathbf{S}(r,0) = \mathbf{0}\,, \tag{3}$$

and the constitutive equation $(1)_2$ implies $S_{rr} = S_{rz} = S_{\theta z} = S_{zz} = 0$ and the meaningful partial differential equation [5]

$$\begin{aligned}&\left(1+\lambda\frac{\partial}{\partial t}\right)\tau(r,t) =\\ &\mu\left(1+\lambda_{\mathrm{r}}\frac{\partial}{\partial t}\right)\left(\frac{\partial}{\partial r}-\frac{1}{r}\right)\omega(r,t)\,,\end{aligned} \tag{4}$$

where $\tau(r,t) = S_{r\theta}(r,t)$ is the non-trivial shear stress. Neglecting body forces, the balance of linear momentum leads to the relevant equation ($\partial_\theta p = 0$ due to the rotational symmetry [5])

$$\rho\frac{\partial\omega(r,t)}{\partial t} = \left(\frac{\partial}{\partial r}+\frac{2}{r}\right)\tau(r,t)\,, \tag{5}$$

where ρ is the constant density of the fluid.

Usually, in the literature, the governing equation for velocity is obtained eliminating $\tau(r,t)$ between (4) and (5). Since our interest here is to solve a motion problem with shear stress on the boundary, we follow [23] and eliminate $\omega(r,t)$ in order to get the governing equation

$$\begin{aligned}&\left(1+\lambda\frac{\partial}{\partial t}\right)\frac{\partial\tau(r,t)}{\partial t} = \nu\left(1+\lambda_{\mathrm{r}}\frac{\partial}{\partial t}\right)\\ &\cdot\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{4}{r^2}\right)\tau(r,t)\,;\;\; r\in(0,R)\,,\;\; t>0\,,\end{aligned} \tag{6}$$

for the shear stress $\tau(r,t)$. Here $\nu = \mu/\rho$ is the kinematic viscosity of the fluid.

The appropriate initial and boundary conditions are

$$\tau(r,0) = \left.\frac{\partial \tau(r,t)}{\partial t}\right|_{t=0} = 0 \ \text{ for } \ r \in [0,R]; \qquad \tau(R,t) = fH(t) \ \text{ for } \ t \geq 0, \tag{7}$$

where $H(t)$ is the Heaviside unit step function and f is constant. In the following the integral transforms technique is used to determine the shear stress distribution resulting from the initial boundary-value problem (6) and (7). The velocity $\omega(r,t)$ is then obtained solving the partial differential equation (5) with the initial condition $(3)_1$.

If $g_{\mathrm{H}}(r_n)$ is the finite Hankel transform of the function $g(r)$, for instance, then [25, Sect. 14.1]

$$g_{\mathrm{H}}(r_n) = \int_0^R r g(r) J_2(r r_n)\,\mathrm{d}r \ \text{ and } \ g(r) = \frac{2}{R^2} \sum_{n=1}^{\infty} \frac{J_2(r r_n)}{[J_2'(R r_n)]^2} g_{\mathrm{H}}(r_n), \tag{8}$$

where $J_2(\cdot)$ denotes the Bessel function of the first kind of second order and the sum is taken over all positive roots r_n of the transcendental equation

$$J_2(Rr) = 0. \tag{9}$$

3. Solution of the Problem

Applying the Laplace transform to (6) and bearing in mind the initial and boundary conditions (7), we find that

$$\bar{\tau}(r,q) = \frac{\nu(1+\lambda_{\mathrm{r}} q)}{q+\lambda q^2} \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2} \right) \bar{\tau}(r,q); \qquad \bar{\tau}(R,q) = \frac{f}{q}, \tag{10}$$

where $\bar{\tau}(r,q)$ is the Laplace transform of $\tau(r,t)$ and q is the transform parameter.

Now, we multiply (10) by $rJ_2(rr_n)$ where r_n are the positive roots of (9), integrate the result from 0 to R and use the identity [25, Sect. 14, Eq. (59)]

$$\int_0^R r J_2(r r_n) \left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} - \frac{4}{r^2} \right) \bar{\tau}(r,q)\,\mathrm{d}r = -R r_n J_2'(R r_n) \bar{\tau}(R,q) - r_n^2 \bar{\tau}_{\mathrm{H}}(r_n,q), \tag{11}$$

in order to obtain

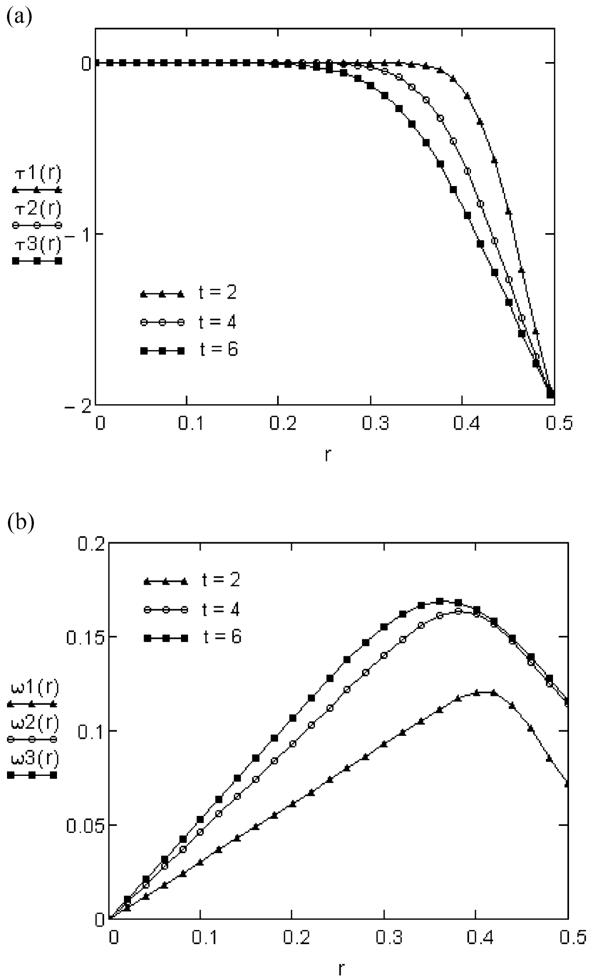


Fig. 1. Profiles of shear stress $\tau(r,t)$ and velocity $\omega(r,t)$ given by (15) and (17) for $R = 0.5$, $f = -2$, $\nu = 0.001188$, $\mu = 1.045$, $\lambda = 2$, $\lambda_{\mathrm{r}} = 1$, and different values of t.

$$\bar{\tau}_{\mathrm{H}}(r_n,q) = -\frac{\nu R f r_n (1+\lambda_{\mathrm{r}} q)}{q\left[\lambda q^2 + q(1+\nu\lambda_{\mathrm{r}} r_n^2) + \nu r_n^2\right]} \cdot J_1(R r_n). \tag{12}$$

In order to present the shear stress $\tau(r,t)$ in a suitable form, we write (12) as a sum, i. e.

$$\bar{\tau}_{\mathrm{H}}(r_n,q) = -\frac{Rf}{r_n q} J_1(R r_n) + \frac{Rf(1+\lambda q)}{r_n\left[\lambda q^2 + q(1+\nu\lambda_{\mathrm{r}} r_n^2) + \nu r_n^2\right]} J_1(R r_n), \tag{13}$$

apply the inverse Laplace and Hankel transforms and use the identity [25, the entry 1 of Table X]:

$$r^2 = -2R\sum_{n=1}^{\infty}\frac{J_2(rr_n)}{r_nJ_1(Rr_n)}. \tag{14}$$

Lengthy but straightforward computations show that

$$\tau(r,t) = \frac{r^2}{R^2}fH(t) + \frac{2}{R}fH(t)\sum_{n=1}^{\infty}\frac{J_2(rr_n)}{r_nJ_1(Rr_n)} \cdot\left[\cosh\left(\frac{b_n}{2\lambda}t\right) + \frac{c_n}{b_n}\sinh\left(\frac{b_n}{2\lambda}t\right)\right]\mathrm{e}^{-\frac{a_n}{2\lambda}t}, \tag{15}$$

where $a_n = 1 + \nu\lambda_r r_n^2$, $b_n = \sqrt{(1+\nu\lambda_r r_n^2)^2 - 4\nu\lambda r_n^2}$, and $c_n = 1 - \nu\lambda_r r_n^2$.

For velocity we introduce (15) into (5) and get

$$\frac{\partial\omega(r,t)}{\partial t} = \frac{4fr}{\rho R^2}H(t) + \frac{2f}{\rho R}H(t)\sum_{n=1}^{\infty}\frac{J_1(rr_n)}{r_nJ_1(Rr_n)} \cdot\left[\cosh\left(\frac{b_n}{2\lambda}t\right) + \frac{c_n}{b_n}\sinh\left(\frac{b_n}{2\lambda}t\right)\right]\mathrm{e}^{-\frac{a_n}{2\lambda}t}. \tag{16}$$

The solution of (16) with the corresponding initial condition $(3)_1$ is given by

$$\omega(r,t) = \frac{4frt}{\rho R^2}H(t) + \frac{2f}{\mu R}H(t)\sum_{n=1}^{\infty}\frac{J_1(rr_n)}{r_n^2J_1(Rr_n)}\left\{1 - \left[\cosh\left(\frac{b_n}{2\lambda}t\right) + \frac{a_n - 2\nu\lambda r_n^2}{b_n}\sinh\left(\frac{b_n}{2\lambda}t\right)\right]\mathrm{e}^{-\frac{a_n}{2\lambda}t}\right\}. \tag{17}$$

A simple analysis shows that $\tau(r,t)$ and $\omega(r,t)$ given by (15) and (17) satisfy both the governing equations (4)–(6) and all imposed initial and boundary conditions.

4. Limiting Cases

4.1. Case $\lambda_r \to 0$ (Maxwell fluids)

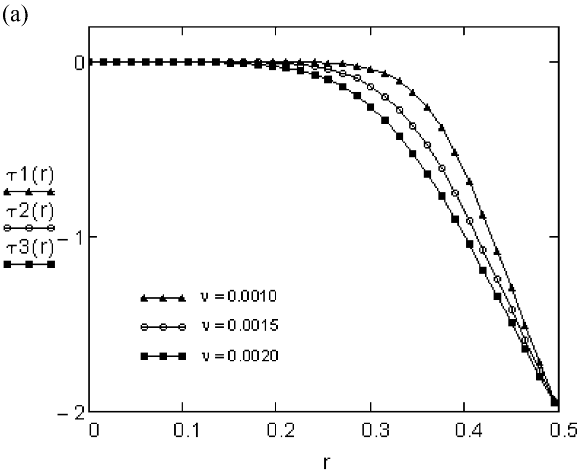
The solutions corresponding to Maxwell fluids performing the same motion, namely

$$\tau_{\mathrm{M}}(r,t) = \frac{r^2}{R^2}fH(t) + \frac{2}{R}fH(t)\sum_{n=1}^{\infty}\frac{J_2(rr_n)}{r_nJ_1(Rr_n)} \cdot\left[\cosh\left(\frac{d_n}{2\lambda}t\right) + \frac{1}{d_n}\sinh\left(\frac{d_n}{2\lambda}t\right)\right]\mathrm{e}^{-\frac{t}{2\lambda}}, \tag{18}$$

$$\omega_{\mathrm{M}}(r,t) = \frac{4frt}{\rho R^2}H(t) + \frac{2f}{\mu R}H(t)\sum_{n=1}^{\infty}\frac{J_1(rr_n)}{r_n^2J_1(Rr_n)} \cdot\left\{1 - \left[\cosh\left(\frac{d_n}{2\lambda}t\right) + \frac{1-2\nu\lambda r_n^2}{d_n}\sinh\left(\frac{d_n}{2\lambda}t\right)\right]\mathrm{e}^{-\frac{t}{2\lambda}}\right\}, \tag{19}$$

are immediately obtained by making $\lambda_r \to 0$ into (15) and (17). Into above relations $d_n = \sqrt{1 - 4\nu\lambda r_n^2}$.

(a)



(b)

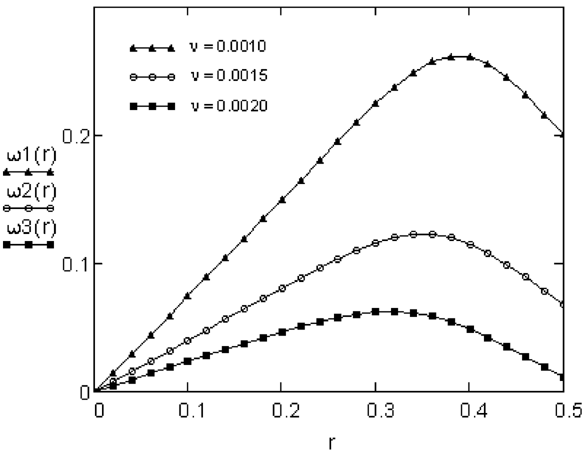


Fig. 2. Profiles of shear stress $\tau(r,t)$ and velocity $\omega(r,t)$ given by (15) and (17) for $R = 0.5$, $f = -2$, $\lambda = 2$, $\lambda_r = 1$, $t = 5$ and different values of ν.

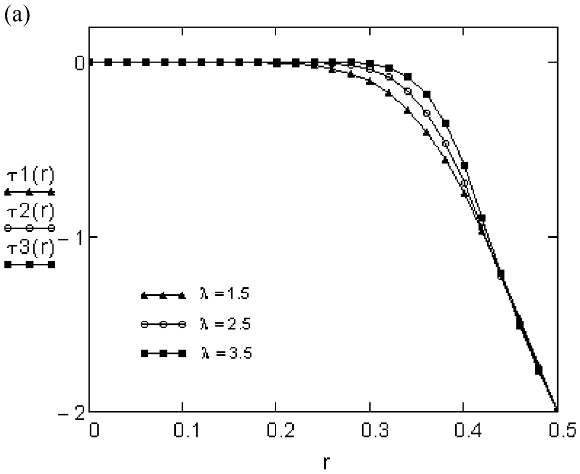
4.2. Case $\lambda \to 0$ (Second-grade fluids)

By now letting $\lambda \to 0$ into (15) and (17), we obtain

$$\tau_{\mathrm{SG}}(r,t) = \frac{r^2}{R^2}fH(t) + \frac{2}{R}fH(t) \cdot\sum_{n=1}^{\infty}\frac{J_2(rr_n)}{r_nJ_1(Rr_n)}\frac{1}{1+\alpha r_n^2}\exp\left(-\frac{\nu r_n^2 t}{1+\alpha r_n^2}\right), \tag{20}$$

$$\omega_{\mathrm{SG}}(r,t) = \frac{4frt}{\rho R^2} + \frac{2f}{\mu R} \cdot\sum_{n=1}^{\infty}\frac{J_1(rr_n)}{r_n^2J_1(Rr_n)}\left[1 - \exp\left(-\frac{\nu r_n^2 t}{1+\alpha r_n^2}\right)\right], \tag{21}$$

(a)



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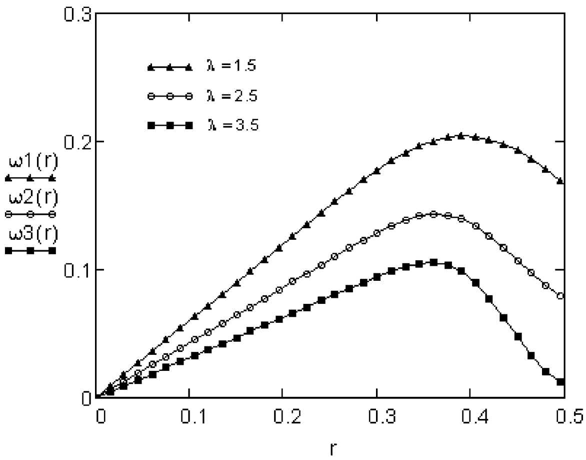

Fig. 3. Profiles of shear stress $\tau(r,t)$ and velocity $\omega(r,t)$ given by (15) and (17) for $R = 0.5$, $f = -2$, $\nu = 0.001188$, $\mu = 1.045$, $\lambda_r = 1$, $t = 5$ and different values of λ.

where $\alpha = \nu\lambda_r$. Direct computations clearly show that $\tau_{SG}(r,t)$ and $\omega_{SG}(r,t)$, given by (20) and (21), satisfy both the initial and boundary conditions and the governing equations (4), (5), and (6) when $\lambda = 0$ and $\lambda_r = \alpha/\nu$.

4.3. Case $\lambda \to 0$, $\lambda_r \to 0$ (Newtonian fluids)

Finally, by making λ_r and $\lambda \to 0$ into (15) and (17) or $\lambda \to 0$ into (18) and (19), as well as $\alpha \to 0$ into (20) and (21), the solutions

$$\tau_N(r,t) = \frac{r^2}{R^2} f + \frac{2}{R} f \sum_{n=1}^{\infty} \frac{J_2(rr_n)}{r_n J_1(Rr_n)} e^{-\nu r_n^2 t}, \tag{22}$$

$$\omega_N(r,t) = \frac{4frt}{\rho R^2} + \frac{2f}{\mu R} \sum_{n=1}^{\infty} \frac{J_1(rr_n)}{r_n^2 J_1(Rr_n)} \left[1 - e^{-\nu r_n^2 t}\right], \tag{23}$$

corresponding to a Newtonian fluid performing the same motion are obtained.

Now, it is worth pointing out that the shear stresses $\tau_{SG}(r,t)$ and $\tau_N(r,t)$ given by (20) and (22) are identical to those obtained in [20, Eqs. (4.7) and (4.11)] while the corresponding expressions for velocity correct the similar results from the same reference.

5. Numerical Results and Conclusions

Generally, the study of a fluid motion as well as the deformation of a solid body can be reduced to an initial and boundary-value problem. Such a problem can be mixed or contains boundary conditions on velocity or shear stress. A fluid motion can be the result of several effects such as the motion of a boundary, wall that applies a shear stress to the fluid or application of a pressure gradient. In the present paper, in order to study the motion of an Oldroyd-B fluid induced by an infinite circular cylinder that applies a constant shear to the fluid, a governing equation for the shear stress $\tau(r,t)$ is developed. This equation, uncommon in the literature, is obtained by eliminating the velocity $\omega(r,t)$ between the motion and constitutive equations. Exact solutions are established both for velocity and shear stress. They are the first exact solutions of this type for Oldroyd-B fluids and can be easy reduced to the similar solutions for Maxwell, second-grade, and Newtonian fluids performing the same motion. The motion in discussion, as it results from (17), (19), (21), and (23), is unsteady. However, for large times, the shear stress distribution within fluid $\tau(r,\infty) = r^2 f/R^2$ is the same both for Newtonian and non-Newtonian fluids.

Now, in order to bring to light some physical insight of present results and to underline the effects of material parameters on the fluid motion, the diagrams of velocity and shear stress against r are presented for different values of t, ν, λ, and λ_r. Figures 1a and 1b contain the diagrams of velocity and shear stress at different times for the same fixed values of material constants. The velocity of the fluid, as well as the shear stress in absolute value, is an increasing function of t. The absolute value of the shear stress, as expected, decreases from its maximum value on the boundary to the zero

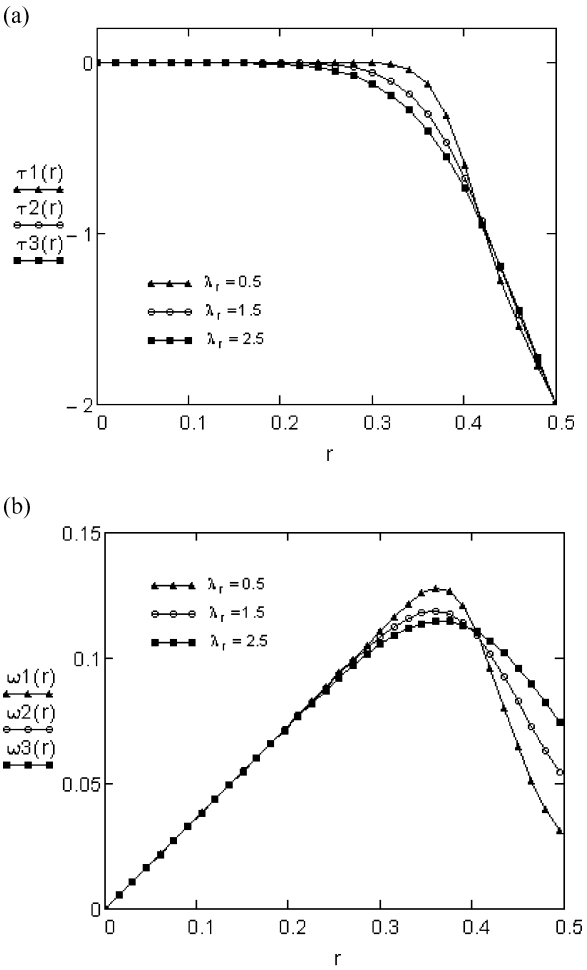

Fig. 4. Profiles of shear stress $\tau(r,t)$ and velocity $\omega(r,t)$ given by (15) and (17) for $R = 0.5$, $f = -2$, $\nu = 0.001188$, $\mu = 1.045$, $\lambda = 3$, $t = 5$ and different values of λ_r.

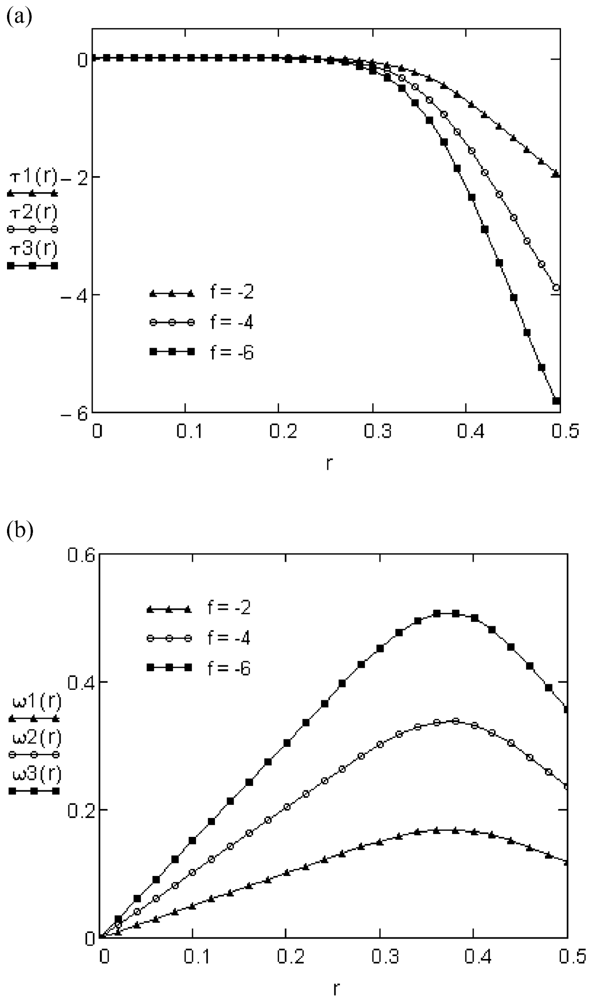

Fig. 5. Profiles of shear stress $\tau(r,t)$ and velocity $\omega(r,t)$ given by (15) and (17) for $R = 0.5$, $\nu = 0.001188$, $\mu = 1.045$, $\lambda = 2$, $\lambda_r = 1$, $t = 5$ and different values of f.

value at the middle of the cylinder. The influence of the rigid boundary on the fluid motion is significantly. It is observed that the velocity increases near the boundary, becomes maximum and then smoothly decreases to the zero value for decreasing r. Its values for each distance from the boundary are always higher for $t = 6$ than for $t = 4$ or $t = 2$. Figure 2 shows the influence of the kinematic viscosity ν on the fluid motion. As expected, the velocity of the fluid decreases while the shear stress in absolute value increases for increasing values of ν.

Figures 3 and 4 illustrate the effects of the relaxation and retardation times λ and λ_r. As expected, the shear stress in absolute values and the fluid velocity are decreasing functions with respect to λ on the whole variation domain of r. Concerning λ_r, there exists a critical point in the neighbourghood of the boundary where the two entities change their monotony. Until the critical point the shear stress in absolute value increases while the fluid velocity decreases for increasing λ_r. Opposite effects appear in the neighbourghood of the boundary. Last, Figure 5 presents the shear stress and velocity profiles against r at three different values of f. As expected, both the shear stress in absolute value and velocity increase for increasing values of the absolute value of f. The shear stress smoothly decreases to zero from its maximum values on the boundary while the velocity, as before, presents over-shoots close to the moving cylinder.

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