Kaluza–Klein Cosmological Model, Strange Quark Matter, and Time-Varying Lambda

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Z. Naturforsch. **69a,** 90 – 96 (2014) / DOI: 10.5560/ZNA.2013-0079
Received May 28, 2013 / revised October 2, 2013 / published online December 18, 2013

In this paper, exact solutions of the Einstein field equations of the Kaluza–Klein cosmological model have been obtained in the presence of strange quark matter. We have considered the time-varying cosmological constant Λ as $\Lambda = \alpha \mathrm{H}^2 + \beta \mathrm{R}^{-2}$, where α and β are free parameters. The solutions are obtained with the help of the equation of state for strange quark matter as per the Bag model, i.e. quark pressure $p = 1/3(\rho - 4B_{\mathrm{C}})$, where B_{C} is Bag's constant. We also discussed the physical implications of the solutions obtained for the model for different types of universes.

Key words: Kaluza–Klein Cosmological Model; Bag Model; Cosmological Constant.
PACS numbers: 98.80-k; 98.80,Hw; 98.80 $\mathrm{V_c}$

1. Introduction

The physical situation prevailing during the early stages of the formation of the universe is still a challenge and an area of major research in cosmology. A lot of efforts are directed in this area, which prompted us to look for dimensions more than space-time $(3+1)$ for the early universe.

The necessity for higher dimensions is thought for the early universe, as it was very small in its early stages. The extra dimensions become compactified and get embedded into four dimensions due to expansion of the universe. Hence, experimental detection of an extra dimension is not possible today due to certain practical limitations, however, its effects can be observed.

Contemporarily, the unification of forces had been worked upon by the research community to find its origin. Kaluza [1] in 1921 and Klein [2] in 1926 independently put forward theories of higher dimensions for the unification of all forces of nature and particle interaction, respectively. A voluminous literature on the Kaluza–Klein (KK) theory of gravitation, its cosmic implications and astrophysical consequences are available now, which has been widely referred to solve issues like accelerated expansion, mystery of dark energy, dark matter, etc.

It is well known that the universe is expanding and also accelerating. The driving force for accelerated expansion is supposed to be dark energy. The relation between extra dimensions with dark energy is enlightened by Gu [3]. Interacting dark energy models with KK cosmology are discussed by Ranjit et al. [4], Chakraborty et al. [5], Sharif and Jawad [6], and Sharif and Khanum [7]. The extra dimension topology and accelerated expansion of the universe have been touched upon by El-Nabulsi [8]. Thus, the KK cosmology and its models have gained importance among the scientific community to learn the secrets of the universe, its behaviour at early times, etc.

In this paper, we examine the KK cosmological model in presence of strange quark matter (SQM) with decaying Lambda. The importance of quark matter lies not only in the structural formation of the universe and, subsequently, its evolution; but since it is a part of dark matter, its importance lies also in the interaction with dark energy. These are hotly debated issues discussed in published literature recently [9 – 11].

Quarks can also be studied with domain walls and strings. Currently, models with quark matter with do-

main walls, strings, etc. have been studied by Adhav and Nimkar [12], Ozel et al. [13], Bali and Pradhan [14], and Yilmaz and Yavuz [15] in different contexts.

Hence, it is necessary to take a look at the role played by strange quark matter in the early stages of the evolution of the universe, since the big bang. It is well known that quark-gluon-plasmas (QGPs) exist since the beginning of the universe.

Since the big bang, the universe has transited through two phases: the first transition occurred at a critical temperature resulting in stable topological defects, while the second transition occurred at the cosmic temperature of the universe $T \sim 200$ Mev. At this temperature, a QGP converted to a hadron gas. The astrophysical consequences of phase transitions have been pointed out by Witten [16] in 1984. The existence of quark matter was first discussed by Itoh [17] and Bodmer [18] in the 1970s and later, Witten [16] proposed that the quark-gluon-hadron gas transition and the conversion of neutron stars into strange stars at ultra-high densities were the two ways for the formation of quark matter. Sagert et al. [19] discussed experimental analysis of explosive astrophysical systems and enlightened quark-hadron phase transitions. Farhi and Jaffe [20], Xu [21], and Lipkin [22] independently reviewed the physical nature of SQM, concluding that it is stable.

Properties of quark matter [u (up), d (down), s (strange) etc. quarks] have been well explained in particle physics where quark matter participates in strong interactions and forms the basic constituent of baryons. Thus, undertaking a study of SQM and quark matter can provide an idea on the structure and the geometry of the universe [23]. A study of quark matter and SQM has been a major area of interest among scientists as it could not only provide information about the early universe, but can also solve the mystery of dark matter. In this context, Virginia Trimble has given an excellent review [24].

In a typical cosmological model with SQM, the quark matter is modelled with the help of a phenomenological Bag model, where the quark confinement has been described as an energy term proportional to the volume [17].

In this model, quarks have been assumed to be degenerate Fermi gases, which exist only in a region of space endowed with a vacuum energy density B_{C} (called the Bag constant). Quark matter consists of massless u, d, and massive s quarks and electrons. In a simplified version of the Bag model, quark density and quark pressure have been related with each other as $p_{\mathrm{q}} = \rho_{\mathrm{q}}/3$, while the total pressure is $p = p_{\mathrm{q}} - B_{\mathrm{C}}$ and the total density $\rho = \rho_{\mathrm{q}} + B_{\mathrm{C}}$. Under these conditions, one obtains the equation of state (EOS) for SQM described as $p = 1/3(\rho - 4B_{\mathrm{C}})$. Experimental results obtained in Brookhaven's relativistic heavy ion collider (BNL-RHIC) laboratory conclude that a quark-gluon-plasma is the perfect fluid, of which quark matter is a basic constituent. Yilmaz and Yavuz [15] and others inferred that the presence of extra dimensions and SQM tended to exert negative pressure with constant density in the early universe. This was concluded to be dark energy.

In fact, the cosmological constant Λ represents dark energy, and it plays an important role. In a recent development in cosmology, it was found that the acceleration of the universe is due to negative pressure which is proportional to the related vacuum density. Present-day astronomical observations [25] indicate that the value is $\leq 10^{-56}\,\mathrm{cm}^2$. But the huge difference between the present small observed value of the cosmological constant and the one calculated by the Glashow–Weinberg–Salam model [26] from particle interaction has been of the order of 10^{50}. This is known as the cosmological constant problem (CCP) and is also a major area of research for many contemporary cosmologists.

Bambi [27] discussed the CCP and SQM and remarked in his article that strange stars, which are thought to consist of SQM, if these exist, can be a good laboratory to bring information about the early universe and their physical conditions during those early stages. It can be considered to investigate the CCP and to test the nature of dark energy.

The time varying cosmological constant Λ was suggested for solving the CCP, as it has been thought that perhaps Λ might had a large value in early universe and decayed with time so as to have the present small value. A decaying Λ has been first explained by Chen and Wu [28] who have suggested that $\Lambda \propto R^{-2}$. Thereafter Sahni and Starobinsky [29], Padmanabhan [30], and Overduin and Cooperstock [31] reviewed cosmological models with time varying Λ in four dimensions in different contexts. Recently, the papers by Khadekar et al. [32–34] dealt with the solutions of the KK cosmological model with different forms of time varying cosmological constants, i.e., $\Lambda \propto \frac{\dot{a}^2}{a^2}$, $\Lambda \propto \frac{1}{a^2}$, $\Lambda \propto \frac{\ddot{a}}{a}$, $\Lambda \propto \rho$; here a is the scale fac-

tor. It has been shown by the authors that these models are dynamically equivalent for a spatially flat universe. El-Nebulsi [35] has discussed a higher-dimensional nonsingular cosmology dominated by a varying cosmological constant. One of the motivations for introducing Λ is to reconcile the age parameters and density parameters with recent observational data. The variation of Λ has also been discussed in literature in different contexts [36–43].

Encouraged by aforementioned facts, we investigated the KK cosmological model in the presence of quark matter with the variation of the cosmological constant $\Lambda = \alpha\frac{\dot{R}^2}{R^2} + \beta\frac{1}{R^2}$ which has been first suggested by Carvalho et al. [44]. Particularly, the present paper is focused on the study of a generalized form of the Kaluza–Klein cosmological model. Here α and β are considered as the dimensionless free parameters. The exact solutions of the Einstein field equations so obtained are applied to study the variation of quark density and Λ for different types of universes. Cosmic implications of the model under consideration are also discussed. The paper is organized in four sections. In Section 2, we first set up field equations and later, in Section 3, found solutions followed by discussion and conclusions in Section 4 and Section 5.

2. Metric and Field Equations

To obtain the solutions of Einstein Field equations let us consider the Kaluza–Klein metric, which is given as follows:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + R^2(t) \cdot \left[\left\{\frac{\mathrm{d}r^2}{(1-kr^2)} + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2\right)\right\}\right] + A^2(t)\,\mathrm{d}\Psi^2 . \tag{1}$$

Also, assume that $\hbar = c = 8\pi G = 1$ in accordance with cosmic principle. $R(t)$ and $A(t)$ are the fourth and fifth dimension scale factors and k is the curvature constant: $k = 0, \pm 1$ for flat, open, and closed model of the universe, respectively. The universe is assumed to be filled with a perfect fluid represented by quark matter. The energy-momentum tensor is given by

$$T_{ij} = (p+\rho)u_i u_j - p g_{ij} , \tag{2}$$

where u_i is the five velocity vector, which satisfies the relation $u_i u^j = 1$. Here p and ρ are quark pressure and quark density, respectively, which are related by EOS per the Bag model given as

$$p = \frac{1}{3}(\rho - 4B_{\mathrm{C}}) , \tag{3}$$

where B_{C} is Bag's constant. The Einstein field equations with time dependent cosmological constant $\Lambda(t)$ are given by

$$R^i_j - \frac{1}{2}g^i_j R = -T^i_j + \Lambda(t) g^i_j . \tag{4}$$

The divergence of Einstein's tensor implies

$$\left(R^i_j - \frac{1}{2}R g^i_j\right)_{;j} = \left(-T^i_j + \Lambda g^i_j\right)_{;j} = 0 . \tag{5}$$

With the help of (1) and (2), we obtain the field equations as

$$2\frac{\ddot{R}}{R} + 2\frac{\dot{R}}{R}\frac{\dot{A}}{A} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + \frac{\ddot{A}}{A} = -p + \Lambda , \tag{6}$$

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}}{R}\frac{\dot{A}}{A} + 3\frac{k}{R^2} = \rho + \Lambda . \tag{7}$$

Conservation of the energy-momentum tensor gives us the following relation:

$$\dot{\rho} + (p+\rho)\left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right) + \dot{\Lambda} = 0 , \tag{8}$$

$$\dot{\rho} + \dot{\Lambda} = -(p+\rho)\left(\frac{3\dot{R}}{R} + \frac{\dot{A}}{A}\right) . \tag{9}$$

According to published literature, exact solutions are obtained using as ansatz the power law equation $A(t) = R^n(t)$ which is also assumed by many researchers [45, and references therein]. The ansatz power law equation is used in view of anisotropy in the universe, despite our assumption of isotropic and homogeneous universe. The expansion scalar θ is proportional to the shear scalar σ, which can be used for a measurement of the anisotropy [46]. This leads to the relation between the metric potentials $R(t)$ and $A(t)$ as $A(t) = R^n(t)$.

It is seen from (6), (7), and (8) that there are three independent equations, and four unknowns R, A, ρ, and Λ. Hence, to solve the field equations, we substitute $A(t) = R^n(t)$ as ansatz in (3), (5), (6), and solve them with (9), getting the following equation:

$$6(n+1)\frac{\ddot{R}\dot{R}}{R^2} - 6(n+1)\frac{\dot{R}^3}{R^3} - 6\frac{k\dot{R}}{R^2} = -\left[\frac{4\rho}{3} - \frac{4}{3}B_{\mathrm{C}}\right](3+n)\frac{\dot{R}}{R} . \tag{10}$$

The above equation can be simplified further by substituting ρ and Λ so as to arrive at a solution and derive other physical parameters. This is explained in the next section.

3. Solutions of Einstein Field Equations with Time-Varying Cosmological Constant

To determine solutions of the Einstein field equations, the assumed form of Λ is $\Lambda = \alpha\frac{\dot{R}^2}{R^2} + \beta\left(\frac{1}{R^2}\right)$, suggested by Carvalho et al. [44]. In this expression, the first term was taken to deal with age and low-density problems while the second term was taken to satisfy the assumption of an isotropic universe.

By substituting ρ from (7), thereafter using Λ, and simplifying (10), the following equation emerges:

$$\frac{\ddot{R}}{R} + \left[\frac{6n^2+15n+9-2\alpha(n+3)}{9(n+1)}\right]\frac{\dot{R}^2}{R^2} + \left[\frac{3k(2n+3)-2\beta(n+3)}{9(n+1)}\right]\frac{1}{R^2} - \frac{2(n+3)}{9(n+1)}B_{\mathrm{C}} = 0\,. \tag{11}$$

Following assumptions have been made in the above equation:

$$m = \frac{6n^2+15n+9-2\alpha(n+3)}{9(n+1)}\,,$$
$$k_1 = \frac{3k(2n+3)-2\beta(n+3)}{9(n+1)}\,,$$
$$k_2 = \frac{2(n+3)}{9(n+1)}B_{\mathrm{C}}\,.$$

Equation (11) now gets simplified as

$$\frac{\ddot{R}}{R} + m\frac{\dot{R}^2}{R^2} + k_1\frac{1}{R^2} - k_2 = 0\,. \tag{12}$$

After some mathematical manipulation the general solution of the above differential equation is

$$\dot{R}^2 = -\frac{k_1}{m} + \frac{k_2}{(m+1)}R^2 + \frac{C_0}{R^{2m}}\,. \tag{13}$$

The above equation is a hyper geometric function but for analytic purpose, we assumed $m = 1$ to get a further simplified solution. The first integral equation of the above differential equation is given by

$$\dot{R}^2 = -k_1 + \frac{k_2}{2}R^2 + \frac{C_0}{R^2}\,, \tag{14}$$

where C_0 is the constant of integration.

The solution of the above equation is arrived by

$$R^2 = \sqrt{\frac{2C_0}{k_2} - \left(\frac{k_1}{k_2}\right)^2}\,\sinh\sqrt{2k_2}(t+c) + \frac{k_1}{k_2}\,. \tag{15}$$

Let $a = \sqrt{\frac{2C_0}{k_2} - \left(\frac{k_1}{k_2}\right)^2}$, $\varphi = \sqrt{2k_2}(t+c)$, and $k_3 = \frac{k_1}{k_2}$, substituting $c = -t_0$ as per present epoch, we can write $\varphi = \sqrt{2k_2}(t-t_0)$.

Consequently, (14) takes the form

$$R(t) = (a\sinh\varphi + k_3)^{\frac{1}{2}}\,. \tag{16}$$

Other physical parameters are calculated as follows:

$$A(t) = (a\sinh\varphi + k_3)^{\frac{n}{2}}\,, \tag{17}$$
$$H(t) = \sqrt{k_2}a\cosh\varphi\,(a\sinh\varphi + k_3)^{-1}\,, \tag{18}$$
$$q(t) = -\left|\frac{R\ddot{R}}{\dot{R}^2}\right| = -\left|\frac{a+k_3\sinh\varphi}{a\cosh^2\varphi}\right|\,, \tag{19}$$
$$\Lambda(t) = \frac{[\alpha]\,k_2a^2\cosh^2\varphi + \beta(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2}\,, \tag{20}$$
$$\rho(t) = \frac{[3(n+1)-\alpha]k_2a^2\cosh^2\varphi + \beta(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2}\,, \tag{21}$$
$$p(t) = \frac{[3(n+1)-\alpha]k_2a^2\cosh^2\varphi + (3k-\beta)(a\sinh\varphi + k_3)}{3(a\sinh\varphi + k_3)^2} - \frac{4B_{\mathrm{C}}}{3}\,. \tag{22}$$

The expressions for quark pressure and quark density can be found out as per the Bag model. We know that $p = p_{\mathrm{q}} - B_{\mathrm{C}}$ and $\rho = \rho_{\mathrm{q}} + B_{\mathrm{C}}$,

$$\rho_{\mathrm{q}}(t) = \frac{[3(n+1)-\alpha]\,k_2a^2\cosh^2\varphi + (3k-\beta)(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2} - B_{\mathrm{C}}\,, \tag{23}$$
$$p_{\mathrm{q}}(t) = \frac{[3(n+1)-\alpha]\,k_2a^2\cosh^2\varphi + (3k-\beta)(a\sinh\varphi + k_3)}{3(a\sinh\varphi + k_3)^2} - \frac{B_{\mathrm{C}}}{3}\,. \tag{24}$$

If $m = 1$ and $k_1 = 1$, we obtain the expressions for α and β as as given below:

$$\alpha = \frac{3(n+1)}{(n+3)}, \quad \beta = \frac{3k(2n+3) - 9(n+1)}{2(n+3)}.$$

Density and pressure for the different values of k are determined as follows:

(i) For flat universe: $k = 0$, $\beta = \frac{-9(n+1)}{2(n+3)}$, $\alpha = \frac{3(n+1)}{(n+3)}$,

$$k_1 = \frac{-2\beta(n+3)}{9(n+1)}, \quad k_2 = \frac{2(n+3)}{9(n+1)} B_C,$$

$$\text{as} \left[k_3 = \frac{k_1}{k_2} = \frac{\beta}{B_C} \right],$$

$$a = \sqrt{\frac{9C_0(n+1)}{2(n+3)B_C} - \left(\frac{\beta}{B_C}\right)^2} = \frac{1}{B_C}\sqrt{\frac{9C_0 B_C(n+1)}{2(n+3)} - \beta^2}, \tag{25}$$

$$\varphi = \sqrt{\frac{4(n+3)B_C}{9(n+1)}}(t - t_0), \tag{26}$$

$$\rho(t) = \frac{\frac{3(n+1)(n+2)}{(n+3)} k_2 a^2 \cosh^2\varphi + \left(\frac{9(n+1)}{2(n+3)}\right)(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2}. \tag{27}$$

(ii) For closed universe: $k = 1$, so, $\alpha = \frac{3(n+1)}{(n+3)}$, $\beta = \frac{-3n}{2(n+3)}$,

$$k_1 = \frac{3(2n+3) - 2\beta(n+3)}{9(n+1)}, \quad k_2 = \frac{2(n+3)}{9(n+1)} B_C,$$

$$\rho(t) = \frac{\frac{3(n+1)(n+2)}{(n+3)} k_2 a^2 \cosh^2\varphi + \left(\frac{9(n+2)}{2(n+3)}\right)(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2}. \tag{28}$$

(iii) For open universe: $k = -1$ then $\alpha = \frac{3(n+1)}{(n+3)}$ and $\beta = \frac{-15n-18}{2(n+3)}$,

$$\rho(t) = \frac{\frac{3(n+1)(n+2)}{(n+3)} k_2 a^2 \cosh^2\varphi + \left(\frac{9n}{2(n+3)}\right)(a\sinh\varphi + k_3)}{(a\sinh\varphi + k_3)^2}. \tag{29}$$

The following section focuses on the dependence on the deceleration parameter, the Hubble parameter, and on the free parameters α and β. From the above equations, it is observed that the free parameters depend upon n which is the index of the power law equation.

4. Discussion

Equations (16) and (17) indicate that the fifth dimension decreases more rapidly than the fourth dimension for $n < 2$. It is also observed that the fifth dimension scale factor is more dominant for small t. Equation (18) suggests that as $t \to t_0$, $H(t)$ tends to reach a constant value. Further, (19) suggests that $q = -1$ if $t \to t_0$, that the universe is accelerating. This condition is also related to eternal inflation of the universe (unending inflation due to expansion of the universe [47]), and its consequences have been pointed out and discussed in literature [48–56]. Eternality of inflation can be related to cosmology to have some parts of the universe to be inflated while others to exit inflation [53]. From (21) and (22), it is observed that for $n = -2$, $\rho(t) = 0$, and so $p = -B_C$. This shows that the universe is expanding. From (20) it is evident that Λ approaches to a small positive value as $t \to \infty$. This is in accordance with recent observed data [57, 58]. From (29) it is clear that for $n = -2$ the density is different for an open universe as compared to that of a flat or closed universe. Furthermore, for $n = 1$ in (10) is same as that obtained by Ozel et al. [13] for a flat universe with $\beta = 0$. The constant integer n here is very important, as it provides information on the nature of extra dimensions. It can be seen from (23) and (24) that quark density and quark pressure depend upon Bag's constant.

Currently, a lot of studies are going on phantom divide crossing. Phantom with $\omega \leq -1$ is dubbed as phantom energy [59]; $\omega = p/\rho$ is a constant in the EOS; $\omega = -1$ is the phantom divide; $\omega > -1$ is the quintessence era while $\omega < -1$ is the phantom era. In present model, if $\omega = -1$, we have $\rho = B_C$, and it is possible to have the phantom divide. For the spatially flat universe from (27), we found that as $t \to t_0$, $\rho = B_C$ if $C_0 = \frac{9(n+1)}{8(n+3)B_C}$.

Thus, it is possible to have a phantom divide crossing in the present model. It is also known from various published literature [60–62] that phantom dark energy models can also explain the accelerated expansion apart from Lambda decaying models. There are various models with $f(R)$ and $f(T)$ (gravity models) explained by Jamil et al. [63], Jamil and Momeni [64], and Momeni and Azadi [65]. These models can explain the accelerated expansion of the universe, however, these models led to a singularity at late times. In this regard, the model with quark matter in $f(R)$ gravity could be useful for the research interest since

quark matter behaves like the phantom type dark matter, and solutions for such models can reveal information about the inflation; and quark matter can be considered as a source of dark energy at the early universe as enlightened by Yilmaz et al. [66]. A modified gravity model at quantum chromodynamic (QCD) scale has also been discussed by Klinkhamer [67]. They claim that their model is advantageous over the Lambda cold dark matter (ΛCDM) model.

5. Conclusions

In this paper, we derived exact solutions in generalized form for the Kaluza–Klein cosmological model in presence of quark matter for a flat, closed, and open universe. Our derived model is a non-singular expanding model, and it generalizes the work done by Ozel et al. [13]. The expressions for density and pressure are observed to be similar. It is also inferred that the model is accelerating at late times. The universe passes from a radiation-dominated phase to a matter-dominated phase, which is the present era, as density as well as pressure decreases exponentially. It is possible to have a phantom divide crossing in the present model, and it can explain eternal inflation. Due to the non-singularity behaviour of our model, it is advantageous over $f(R)$ and $f(T)$ models.

Acknowledgements

Authors are thankful to IUCAA (India), IIT (Mumbai, India), and TIFR (India) for providing library facilities. Authors are also grateful to the referees for their valuable suggestions.

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