Influence of Heat and Mass Transfer on the Peristaltic Transport of a Phan-Thien–Tanner Fluid

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In this paper, we discuss the effects of heat and mass transfer on the peristaltic flow in the presence of an induced magnetic field. Constitutive equations of a Phan-Thien–Tanner fluid are utilized in the mathematical description. Mathematical modelling is based upon the laws of mass, linear momentum, energy, and concentration. Relevant equations are simplified using long wavelength and low Reynolds number assumptions. A series solution is presented for small Weissenberg number. Variations of emerging parameters embedded in the flow system are discussed.

Key words: Heat and Mass Transfer; Phan-Thien–Tanner (PTT) Fluid; Channel.

1. Introduction

During the past four decades there is an increasing interest of the researchers in peristaltic flows. This is in view of extensive applications of such flows in physiology and industry. Many investigations [1 – 15] here examined the peristaltic flow of viscous and non-Newtonian fluids in symmetric/asymmetric channels under varied assumptions of long wavelength, small wave number, small amplitude ratio, low Reynolds number etc.

Despite an existence of large body of literature on the peristaltic flows, not much has been examined on the peristalsis with heat transfer characteristics. Mekheimer and Abd elmaboud [16] analyzed the magnetohydrodynamic (MHD) viscous flow and heat transfer characteristics in a vertical annulus. The interaction of peristaltic flow of a viscous fluid and heat transfer in a vertical porous annulus region is examined by Vajravelu et al. [17]. Hayat et al. made significant contributions on this topic in the studies [18 – 21] for MHD flows. Srinivas and Kothandapani [22] discussed the peristaltic transport of a viscous fluid in an asymmetric channel. Ogulu [23] studied the heat and mass transfer effects by considering blood as a MHD fluid.

The purpose of present attempt is to put forward the analysis of peristaltic flows with heat and mass transfer. Therefore, this article describes the heat and mass transfer effects on the peristaltic flow of a MHD non-Newtonian fluid. Constitutive equations of a Phan-Thien–Tanner (PTT) fluid are taken into consideration. The paper is arranged as follows. In Section 2, we present the basic equations. The problem formulation is given in Section 3. Series solution for small Weissenberg number are presented in Section 4. Section five comprises the interpretation of graphical results.

2. Basic Equations

The constitutive equations of a PTT fluid are given by [24, 25]

$$\mathbf{T} = -p\mathbf{I} + \tau\,, \tag{1}$$

$$f(\mathrm{tr}(\tau))\tau + \kappa\tau^{\nabla} = 2\mu\mathbf{D}\,, \tag{2}$$

$$\tau^{\nabla} = \frac{\mathrm{d}\tau}{\mathrm{d}t} - \tau\cdot\mathbf{L}^{*} - \mathbf{L}\cdot\tau\,, \tag{3}$$

$$\mathbf{L} = \mathrm{grad}\,\mathbf{V}\,.$$

In the above equations $\mathbf{V}, \mathbf{I}, p, \mathbf{T}, \mu, \tau, \mathbf{D}, \kappa, \mathrm{d}/\mathrm{d}t, \mathrm{tr}, \tau^{\nabla}$ and * indicate the velocity, identity tensor, pressure, Cauchy stress tensor, dynamic viscosity, an extra-stress tensor, deformation-rate tensor, relaxation time, material derivative, trace, Oldroyd's upper-convected derivative, and asterisk, respectively.

In the linearized PTT fluid model, the function f satisfies the expression

$$f(\mathrm{tr}(\tau)) = 1 + \frac{\varepsilon\kappa}{\mu}\mathrm{tr}(\tau)\,. \tag{4}$$

When $\varepsilon = 0$, then above expression holds for a Maxwell fluid. The Maxwell relations are

$$\begin{aligned} &\nabla\cdot\mathbf{E} = 0\,,\ \nabla\cdot\mathbf{H} = 0\,,\\ &\nabla\times\mathbf{E} = -\mu_\mathrm{e}\frac{\partial\mathbf{H}}{\partial t}\,,\ \nabla\times\mathbf{H} = \mathbf{J}\,, \end{aligned} \tag{5}$$

and

$$\mathbf{J} = \sigma\left(\mathbf{E} + \mu_\mathrm{e}\left(\mathbf{V}\times\mathbf{H}\right)\right). \tag{6}$$

Note that the displacement current is neglected and $\mathbf{J}, \mu_\mathrm{e}, \sigma, \mathbf{E}$, and $\mathbf{H}$ denote the electric current density, the magnetic permeability, the electrical conductivity, the electric field, and the magnetic field, respectively.

3. Mathematical Formulation

We consider a MHD PTT fluid in a planar channel of uniform thickness $2a$. A sinusoidal wave of velocity c propagates on the non-conducting channel walls. We select rectangular coordinates $(\bar{X},\bar{Y})$ such that $\bar{X}$ is in the direction of wave propagation and $\bar{Y}$ transverse to it. A constant magnetic field of strength H_0 acts in the transverse direction resulting in an induced magnetic field $\mathbf{H}\left(\bar{h}_{\bar{x}}(\bar{X},\bar{Y},\bar{t}), \bar{h}_{\bar{y}}(\bar{X},\bar{Y},\bar{t}), 0\right)$. The total magnetic field is $\mathbf{H}^+\left(\bar{h}_{\bar{x}}(\bar{X},\bar{Y},\bar{t}), H_0 + \bar{h}_{\bar{y}}(\bar{X},\bar{Y},\bar{t}), 0\right)$. The considered wave shape is represented by the expression

$$\bar{h}(\bar{X},\bar{t}) = a + b\sin\left(\frac{2\pi}{\lambda}(\bar{X} - c\bar{t})\right). \tag{7}$$

Here λ is the wavelength, a indicates the channel half width, b the wave amplitude, c the wave speed, and $\bar{t}$ the time. The velocity field for two-dimensional flow is written as

$$\mathbf{V} = [\bar{U}(\bar{X},\bar{Y},\bar{t}), \bar{V}(\bar{X},\bar{Y},\bar{t}), 0]. \tag{8}$$

The transformations between the laboratory $(\bar{X},\bar{Y})$ and wave $(\bar{x},\bar{y})$ frames are related by the following expressions:

$$\begin{aligned} &\bar{x} = \bar{X} - c\bar{t}\,,\ \bar{y} = \bar{Y}\,,\\ &\bar{u}(\bar{x},\bar{y}) = \bar{U} - c\,,\ \bar{v}(\bar{x},\bar{y}) = \bar{V}\,, \end{aligned} \tag{9}$$

in which $(\bar{U},\bar{V})$ and $(\bar{u},\bar{v})$ are the velocity components in the laboratory and wave frames, respectively. The fundamental equations which lead the mathematical formulation are:

Continuity equation

$$\nabla\cdot\mathbf{V} = 0\,. \tag{10}$$

Equation of motion

$$\begin{aligned} \rho\frac{\mathrm{d}\mathbf{V}}{\mathrm{d}t} &= \mathrm{div}\mathbf{T} + \mu_\mathrm{e}\left(\nabla\times\mathbf{H}^+\right)\times\mathbf{H}^+ \\ &= \mathrm{div}\mathbf{T} + \mu_\mathrm{e}\left[\left(\mathbf{H}^+\cdot\nabla\right)\mathbf{H}^+ - \frac{\nabla\mathbf{H}^{+^2}}{2}\right]. \end{aligned} \tag{11}$$

Energy equation

$$\rho C_\mathrm{p}\frac{\mathrm{d}T}{\mathrm{d}t} = \kappa\nabla^2 T + \mathbf{T}\cdot\mathbf{L}\,. \tag{12}$$

Concentration equation

$$\frac{\mathrm{d}C}{\mathrm{d}t} = D\nabla^2 C + \frac{DK_\mathrm{T}}{T_\mathrm{m}}\nabla^2 T\,. \tag{13}$$

Induction equation

$$\frac{\mathrm{d}\mathbf{H}^+}{\mathrm{d}t} = \nabla\times\left(\mathbf{V}\times\mathbf{H}^+\right) + \frac{1}{\varsigma}\nabla^2\mathbf{H}^+\,. \tag{14}$$

In above equations $\varsigma = \sigma\mu_\mathrm{e}$ is the magnetic diffusivity, C_p specific heat at constant pressure, T the temperature, D the coefficient of mass diffusivity, T_m the mean temperature, K_T the thermal diffusion ratio, C the concentration, and κ the thermal conductivity.

The resulting two-dimensional equations in the wave frame are

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0\,, \tag{15}$$

$$\begin{aligned} &\rho\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{u} + \frac{\partial\bar{p}}{\partial\bar{x}} = \frac{\partial\bar{\tau}_{xx}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{xy}}{\partial\bar{y}} \\ &-\frac{\mu_\mathrm{e}}{2}\left(\frac{\partial H^{+^2}}{\partial\bar{x}}\right) + \mu_\mathrm{e}\left(\bar{h}_{\bar{x}}\frac{\partial\bar{h}_x}{\partial\bar{x}} + \bar{h}_{\bar{y}}\frac{\partial\bar{h}_x}{\partial\bar{y}} + H_0\frac{\partial\bar{h}_x}{\partial\bar{y}}\right), \end{aligned} \tag{16}$$

$$\begin{aligned} &\rho\left(\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{v} + \frac{\partial\bar{p}}{\partial\bar{y}} = \frac{\partial\bar{\tau}_{yx}}{\partial\bar{x}} + \frac{\partial\bar{\tau}_{yy}}{\partial\bar{y}} \\ &-\frac{\mu_\mathrm{e}}{2}\left(\frac{\partial H^{+^2}}{\partial\bar{y}}\right) + \mu_\mathrm{e}\left(\bar{h}_{\bar{x}}\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{x}} + \bar{h}_{\bar{y}}\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{y}} + H_0\frac{\partial\bar{h}_{\bar{y}}}{\partial\bar{y}}\right), \end{aligned} \tag{17}$$

$$\rho C_\mathrm{p}\left[\bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right]\bar{T} = \kappa\left[\frac{\partial^2\bar{T}}{\partial\bar{x}^2} + \frac{\partial^2\bar{T}}{\partial\bar{y}^2}\right] \tag{18}$$

$$+\tau_{xx}\frac{\partial \bar{u}}{\partial \bar{x}}+\tau_{yy}\frac{\partial \bar{v}}{\partial y}+\tau_{xy}\left[\frac{\partial \bar{v}}{\partial \bar{x}}+\frac{\partial \bar{u}}{\partial y}\right],$$

$$\left[\bar{u}\frac{\partial}{\partial \bar{x}}+\bar{v}\frac{\partial}{\partial \bar{y}}\right]\bar{C}=D\left[\frac{\partial^2 \bar{C}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{C}}{\partial \bar{y}^2}\right] \tag{19}$$

$$+\frac{DK_{\mathrm{T}}}{T_{\mathrm{m}}}\left[\frac{\partial^2 \bar{T}}{\partial \bar{x}^2}+\frac{\partial^2 \bar{T}}{\partial \bar{y}^2}\right]$$

$$f\bar{\tau}_{xx}+\kappa\left(\bar{u}\frac{\partial \bar{\tau}_{xx}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\tau}_{xx}}{\partial \bar{y}}-2\frac{\partial \bar{u}}{\partial \bar{x}}\bar{\tau}_{xx}-2\frac{\partial \bar{u}}{\partial \bar{y}}\bar{\tau}_{xy}\right) \tag{20}$$

$$=2\mu\frac{\partial \bar{u}}{\partial \bar{x}},$$

$$f\bar{\tau}_{yy}+\kappa\left(\bar{u}\frac{\partial \bar{\tau}_{yy}}{\partial \bar{x}}+\bar{v}\frac{\partial \bar{\tau}_{yy}}{\partial \bar{y}}-2\frac{\partial \bar{v}}{\partial \bar{x}}\bar{\tau}_{yx}-2\frac{\partial \bar{v}}{\partial \bar{y}}\bar{\tau}_{yy}\right) \tag{21}$$

$$=2\mu\frac{\partial \bar{v}}{\partial \bar{y}},$$

$$f\bar{\tau}_{zz}+\kappa\left(u\frac{\partial \bar{\tau}_{zz}}{\partial \bar{x}}+v\frac{\partial \bar{\tau}_{zz}}{\partial \bar{y}}\right)=0, \tag{22}$$

$$f\bar{\tau}_{xy}+\kappa\left(u\frac{\partial \bar{\tau}_{xy}}{\partial \bar{x}}+v\frac{\partial \bar{\tau}_{xy}}{\partial \bar{y}}-\frac{\partial \bar{v}}{\partial \bar{x}}\bar{\tau}_{xx}\right. \tag{23}$$

$$\left.-\frac{\partial \bar{v}}{\partial \bar{y}}\bar{\tau}_{xy}-\frac{\partial \bar{u}}{\partial \bar{x}}\bar{\tau}_{xy}-\frac{\partial \bar{u}}{\partial \bar{y}}\bar{\tau}_{yy}\right)=\mu\left(\frac{\partial \bar{u}}{\partial \bar{y}}+\frac{\partial \bar{v}}{\partial \bar{x}}\right),$$

$$f=1+\frac{\varepsilon\kappa}{\mu}\left(\bar{\tau}_{xx}+\bar{\tau}_{xy}+\bar{\tau}_{zz}\right). \tag{24}$$

In order to proceed with dimensionless variables, we introduce

$$\mathrm{We}=\frac{kc}{a},\ x=\frac{\bar{x}}{\lambda},\ y=\frac{\bar{y}}{a},\ t=\frac{c\bar{t}}{\lambda},\ p=\frac{a^2\bar{p}}{c\lambda\mu},$$
$$M^2=\mathrm{Re}S^2R_{\mathrm{m}},\ \delta=\frac{a}{\lambda},\ \tau_{ij}=\frac{a\bar{\tau}_{ij}}{\mu c}\ (\text{for } i,j=1,2,3),$$
$$u=\frac{\bar{u}}{c},\ v=\frac{\bar{v}}{c},\ \mathrm{Re}=\frac{ca\rho}{\mu},\ R_{\mathrm{m}}=\sigma\mu_{\mathrm{e}}ac,$$
$$S=\frac{H_0}{c}\sqrt{\frac{\mu_{\mathrm{e}}}{\rho}},\ \phi=\frac{\bar{\phi}}{H_0a},\ \bar{h}_{\bar{x}}=\bar{\phi}_{\bar{y}},\ \bar{h}_{\bar{y}}=-\bar{\phi}_{\bar{x}},$$
$$p_{\mathrm{m}}=p+\frac{1}{2}\mathrm{Re}\delta\frac{\mu_{\mathrm{e}}(H^+)^2}{\rho c^2},\ E=\frac{-E}{cH_0\mu_{\mathrm{e}}},$$
$$E_1=\frac{c^2}{C_{\mathrm{p}}T_1},\ \mathrm{Pr}=\frac{\mu C_{\mathrm{p}}}{\kappa},\ \theta_{\mathrm{temp}}=\frac{\bar{T}}{T_1},\ \phi_{\mathrm{con}}=\frac{\bar{C}}{C_1},$$
$$\mathrm{Sc}=\frac{\mu}{\rho D},\ \mathrm{Sr}=\frac{\rho T_0DK_{\mathrm{T}}}{\mu T_{\mathrm{m}}C_1},\ \mathrm{Br}=E_1\mathrm{Pr}, \tag{25}$$

in which $E_1, \mathrm{Pr}, \mathrm{Sc}, \mathrm{Sr}, \delta, \mathrm{We}, \mathrm{Re}, R_{\mathrm{m}}, S$, and M are Eckert, Prandtl, Schmidt, Soret, wave, Weissenberg, Reynolds, magnetic Reynolds, Stommer, and Hartman numbers, respectively. Here p_{m} is the total pressure which is a sum of ordinary and magnetic pressures, E is the electric field strength, Ψ is the stream function, and ϕ is the magnetic force function. Moreover T_1 and C_1 are temperature and concentration at $y=h$, respectively.

Equation (8) in dimensionless variables then can be written as

$$h=\frac{\bar{h}}{a}=1+\alpha\sin(2\pi x), \tag{26}$$

in which the amplitude ratio α is equal to b/a. Writing

$$u=\frac{\partial\Psi}{\partial y},\ v=-\delta\frac{\partial\Psi}{\partial x},$$
$$h_x=\frac{\partial\phi}{\partial y},\ h_y=-\delta\frac{\partial\phi}{\partial x}, \tag{27}$$

and using long wavelength approach, (16) is automatically satisfied, and (15)–(25) in their reduced form are given by

$$\frac{\partial p}{\partial x}=\frac{\partial\tau_{xy}}{\partial y}+\mathrm{Re}\,S^2\frac{\partial^2\phi}{\partial y^2}, \tag{28}$$

$$\frac{\partial p}{\partial y}=0, \tag{29}$$

$$\frac{\partial^2\theta_{\mathrm{temp}}}{\partial y^2}+\mathrm{Br}\psi_{yy}\tau_{xy}=0, \tag{30}$$

$$\frac{\partial^2\phi_{\mathrm{con}}}{\partial y^2}=-\mathrm{Sc}\,\mathrm{Sr}\frac{\partial^2\theta_{\mathrm{temp}}}{\partial y^2}, \tag{31}$$

$$E=\frac{\partial\Psi}{\partial y}+\frac{1}{R_{\mathrm{m}}}\frac{\partial^2\phi}{\partial y^2}, \tag{32}$$

$$f\tau_{xx}=2\mathrm{We}\frac{\partial^2\Psi}{\partial y^2}\tau_{xy}, \tag{33}$$

$$f\tau_{yy}=0=f\tau_{zz}=0, \tag{34}$$

$$f\tau_{xy}=-\mathrm{We}\frac{\partial^2\Psi}{\partial y^2}\tau_{yy}+\frac{\partial^2\Psi}{\partial y^2}, \tag{35}$$

and (29) shows that $p\neq p(y)$ and therefore $p=p(x)$.

The dimensionless boundary conditions are

$$\Psi=0,\ \frac{\partial^2\Psi}{\partial y^2}=0,\ \frac{\partial\phi}{\partial y}=0,$$
$$\frac{\partial\phi_{\mathrm{con}}}{\partial y}=0,\ \frac{\partial\theta_{\mathrm{temp}}}{\partial y}=0\ \text{ at } y=0,$$
$$\Psi=F,\ \frac{\partial\Psi}{\partial y}=-1,\ \phi=0,\ \theta_{\mathrm{temp}}=1, \tag{36}$$
$$\frac{\partial}{\partial y}\phi_{\mathrm{con}}=1\ \text{ at } y=h,$$

$$F=\int_0^h\frac{\partial\Psi}{\partial y}\,\mathrm{d}y. \tag{37}$$

By (34), we have $\tau_{yy} = \tau_{zz} = 0$ and thus the trace of the stress tensor becomes τ_{xx}. Integration of (28) after using (29) subjected to the boundary condition $\tau_{xy} = 0$ at $y = 0$ (the symmetry line) gives

$$\tau_{xy} = y\frac{\mathrm{d}p}{\mathrm{d}x} - M^2(Ey - \Psi). \tag{38}$$

From (34) and (35), one obtains

$$\tau_{xx} = 2\mathrm{We}\,\tau_{xy}^2. \tag{39}$$

Due to (27), (34), and (39), we have

$$\frac{\partial^2\Psi}{\partial y^2} = \tau_{xy} + 2\varepsilon\mathrm{We}^2\tau_{xy}^3. \tag{40}$$

Upon making use of (38) into (40), one arrives at

$$\frac{\partial^2\Psi}{\partial y^2} = y\frac{\mathrm{d}p}{\mathrm{d}x} - M^2(Ey - \Psi) + 2\varepsilon\mathrm{We}^2\left(y\frac{\mathrm{d}p}{\mathrm{d}x} - M^2(Ey - \Psi)\right)^3. \tag{41}$$

4. Perturbation Solution

For a perturbation solution, the flow quantities in terms of We can be written as follows:

$$\Psi = \Psi_0 + \mathrm{We}^2\Psi_1 + O(\mathrm{We})^4, \quad \phi = \phi_0 + \mathrm{We}^2\phi_1 + O(\mathrm{We})^4, \tag{42}$$

$$F = F_0 + \mathrm{We}^2F_1 + O(\mathrm{We})^4, \quad \theta_{\mathrm{tem}} = \theta_{0\mathrm{temp}} + \mathrm{We}^2\theta_{1\mathrm{temp}} + O(\mathrm{We})^4, \tag{43}$$

$$p = p_0 + \mathrm{We}^2p_1 + O(\mathrm{We})^4, \quad \phi_{\mathrm{con}} = \phi_{0\mathrm{con}} + \mathrm{We}^2\phi_{1\mathrm{con}} + O(\mathrm{We})^4. \tag{44}$$

Invoking above expressions into (30), (31), (36), and (41), comparing terms of like powers of We^2, then solving the resulting zeroth- and first-order system and using

$$F_0 = F - \mathrm{We}^2F_1, \tag{45}$$

we obtain analytic expressions for stream function and pressure gradient.

$$\Psi = \frac{(\cosh(3My) - \sinh(3My))(L_1(y) + L_2(y))}{(1 + hM + (-1 + hM)(\cosh(2Mh) + \sinh(2Mh)))} + \mathrm{We}^2 \cdot \frac{[\{L_3(y) + L_4(y) - L_5(y) - L_6(y) + L_7(y) - L_8(y) - L_9(y) + L_{10}(y) + L_{11}(y) + L_{12}(y) + L_{13}(y) + L_{14}(y)\}\{(F+h)^3M^4\varepsilon(\cosh(3My) + \sinh(3My))\}]}{4(1 + hM + (-1 + hM)(\cosh(2Mh) + \sinh(2Mh)))^4}, \tag{46}$$

$$\frac{\mathrm{d}p}{\mathrm{d}x} = -\frac{M^2((F - hE)M\cosh(Mh) + (1 + E)\sinh(Mh))}{hM\cosh(Mh) - \sinh(Mh)} + \mathrm{We}^2 \cdot \frac{\left[M^2\left\{2L(1 + hM + (-1 + hM)C_8)^4 + 2(F+h)^3M^5\varepsilon(\cosh(4Mh) + \sinh(4Mh))(12hM - 8\sinh(2Mh) + \sinh(4Mh))\right\}\right]}{2(1 + hM + (-1 + hM)C_8)^4}. \tag{47}$$

Utilizing expressions of Ψ in (30)–(32), one can obtain the purterbed expressions of $\theta_{\mathrm{tem}}, \phi_{\mathrm{con}}$ and ϕ. Here the involved C_i $(i = 1-8)$ and L_i $(i = 1-14)$ are obtained by simple algebraic computations.

The heat transfer coefficient Z at the wall, the dimensionless axial induced magnetic field h_x, current density J_z, pressure rise ΔP_λ, and friction force F_λ are defined as

$$h_x = \frac{\partial\phi}{\partial y}, \quad J_z = -\frac{\partial^2\phi}{\partial y^2}, \quad \Delta P_\lambda = \int_0^1 \frac{\mathrm{d}p}{\mathrm{d}x}\,\mathrm{d}x, \quad F_\lambda = \int_0^1 h\left(-\frac{\mathrm{d}p}{\mathrm{d}x}\right)\mathrm{d}x, \quad Z = h_x\theta_{\mathrm{ytem}}(h). \tag{48}$$

5. Discussion of Graphs

This section discusses the influence of various parameters (i.e Brinkman number Br, extensional param-

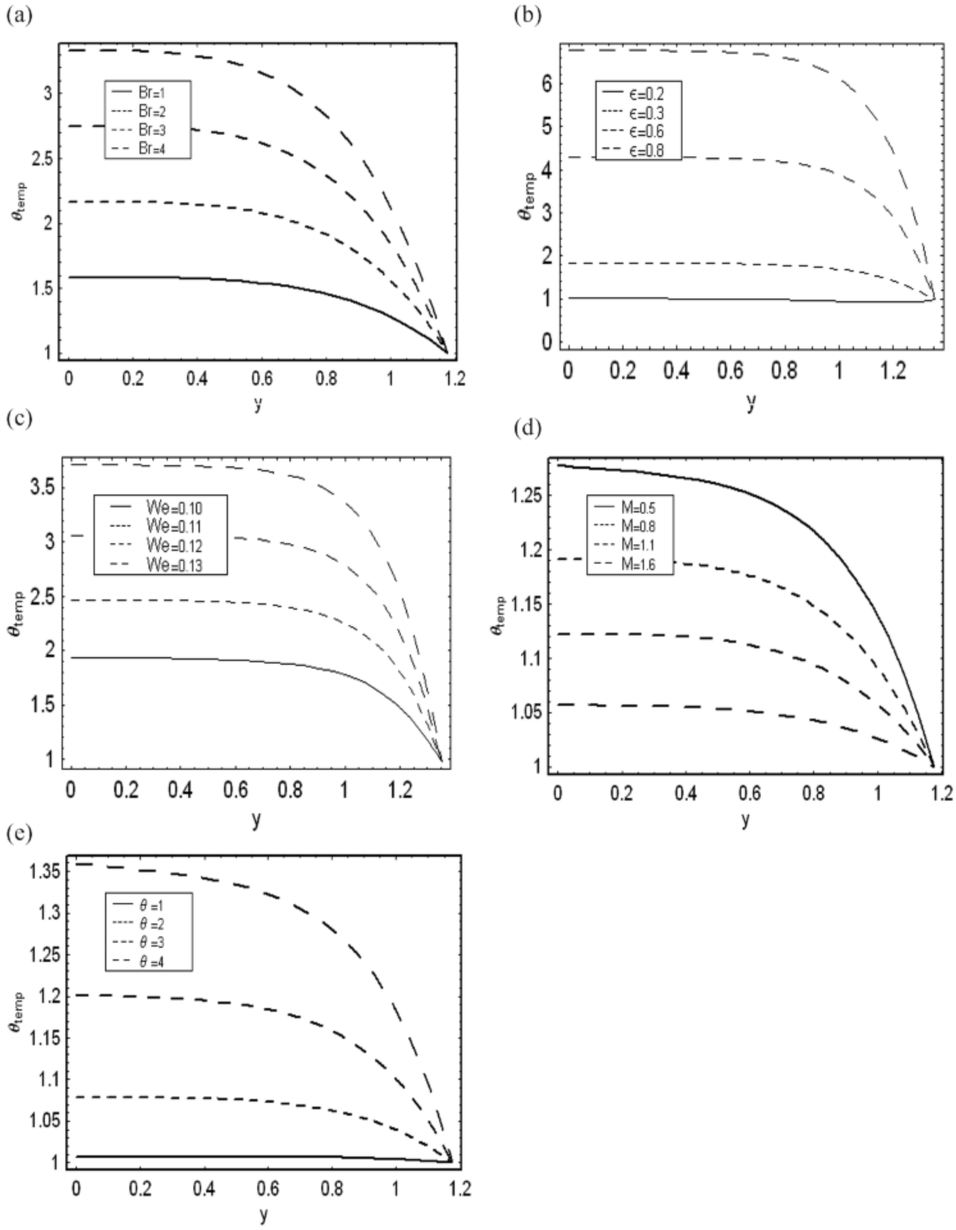

Fig. 1. (a) Temperature distribution θ_{temp} versus y for $E = 1, M = 2, \varepsilon = 0.3, \alpha = 0.6, \theta = 1.5, \text{We} = 0.01$, and $x = 0.1$. (b) Temperature distribution θ_{temp} versus y for $E = 1, M = 5.2, \text{Br} = 0.1, \alpha = 0.6, \theta = 2, \text{We} = 0.08$, and $x = 0.1$. (c) Temperature distribution θ_{temp} versus y for $E = 1, M = 5.2, \text{Br} = 0.1, \alpha = 0.6, \theta = 2, \varepsilon = 0.2$, and $x = 0.1$. (d) Temperature distribution θ_{temp} versus y for $M = 2, \text{Br} = 0.3, \alpha = 0.6, \theta = 1.5, \varepsilon = 0.3, x = 0.1$, and $\text{We} = 0.03$. (e) Temperature distribution θ_{temp} versus y for $M = 2, \alpha = 0.2, \text{Br} = 0.1, \varepsilon = 0.3, x = 0.1, \text{We} = 0.03$, and $E = 1$.

eter ε, Weissenberg number We, and flow rate θ, amplitude ratio α, Schmidt number Sc, Hartman number M, and magnetic Reynolds number R_{m}) involved on the temperature θ_{temp}, heat transfer coefficient Z, mass concentration ϕ_{con}, magnetic force function ϕ, current density J_z, and axial induced magnetic field h_x. For this purpose, Figures 1–3 are sketched. Plots for the stream function ψ, pressure gradient $\mathrm{d}p/\mathrm{d}x$, pressure

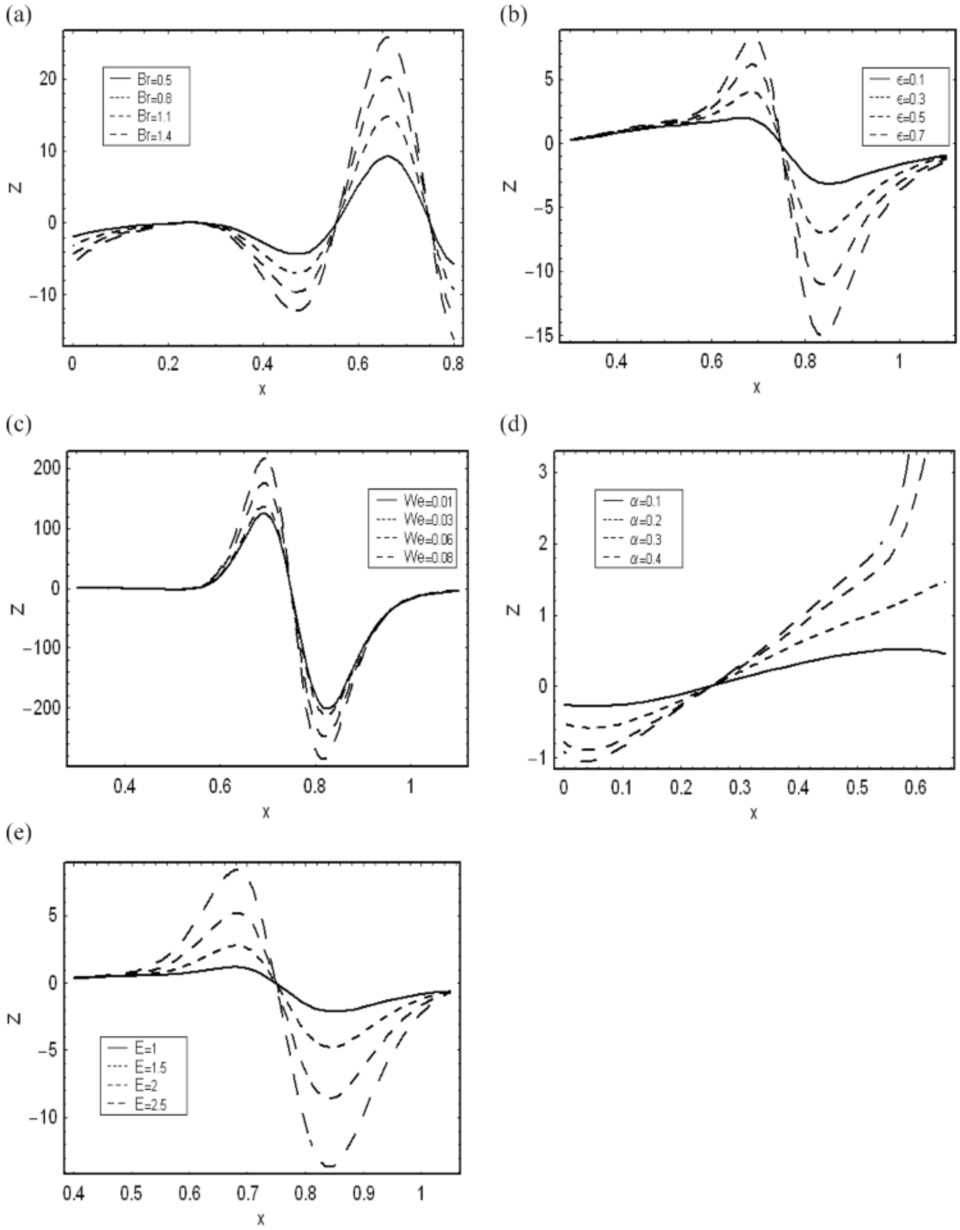

Fig. 2. (a) Heat coefficient Z versus x for $M = 2.5, \alpha = 0.2, \theta = 1.5, \varepsilon = 0.3, \mathrm{We} = 0.03$, and $E = 1$. (b) Heat coefficient Z versus x for $M = 2, \alpha = 0.2, \theta = 1.5, \mathrm{Br} = 1, \mathrm{We} = 0.03$, and $E = 1$. (c) Heat coefficient Z versus x for $M = 1, \alpha = 0.2, \theta = 2, \mathrm{Br} = 0.1, \mathrm{Br} = 4, \varepsilon = 0.3$, and $E = 1$. (d) Heat coefficient Z versus x for $M = 1, \alpha = 0.2, \theta = 2, \mathrm{Br} = 4, \mathrm{We} = 0.03, \mathrm{Br} = 4, \varepsilon = 0.3$, and $E = 5$. (e) Heat coefficient Z versus y for $M = 1, \alpha = 0.3, \theta = 4.5, Br = 0.3, Br = 4, \varepsilon = 0.3$, and $\mathrm{We} = 0.05$.

rise ΔP_λ, and frictional forces F_λ are displayed for the effects of Hartmann number M.

The variation of Br, ε, We, θ, and M on the temperature distribution is shown in Figure 1. One sees that the dimensionless temperature profiles are almost parabolic in nature. The Brinkman number demonstrates the role of viscous dissipation. The temperature distribution is an increasing function of the Brinkman number Br, extensional parameter ε, Weissenberg number We, and flow rate θ

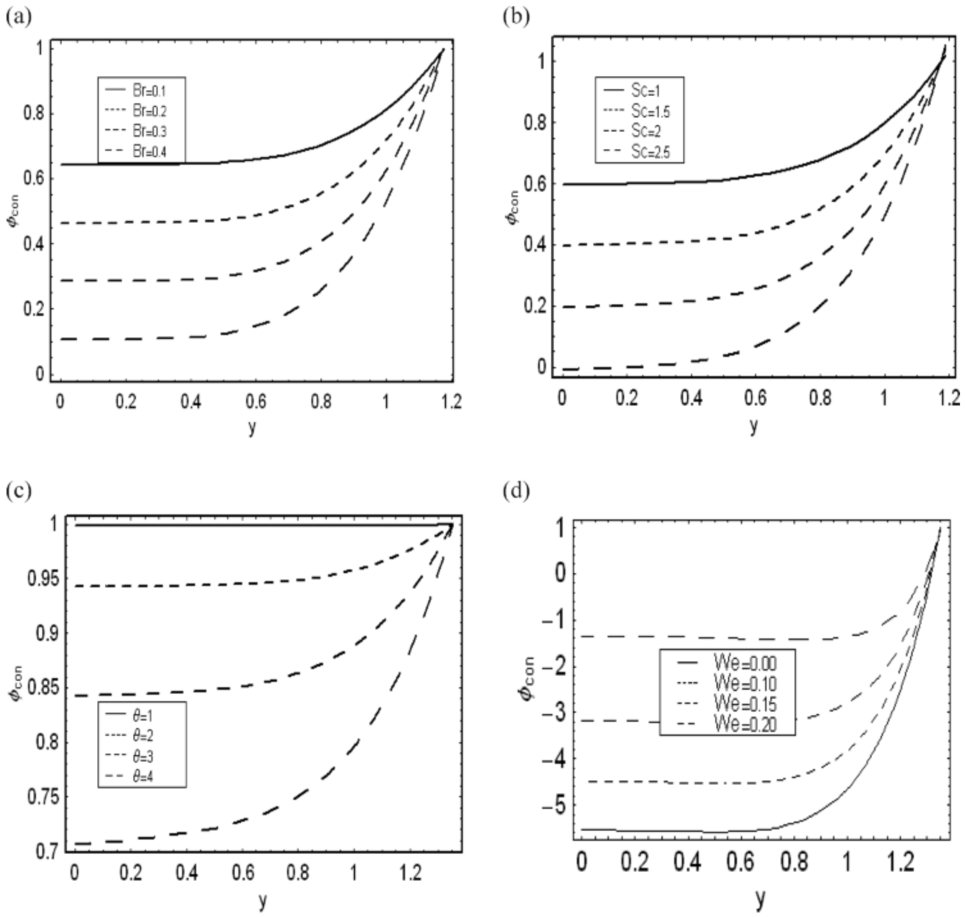

Fig. 3. (a) Concentration distribution ϕ_{con} versus y for $M = 3, \alpha = 0.2, \theta = 2.5, x = 0.3, \text{Sc} = 3, \text{Sr} = 3, \varepsilon = 0.3, \text{We} = 0.03$, and $E = 1$. (b) Concentration distribution ϕ_{con} versus y for $M = 3, \alpha = 0.2, \theta = 2.5, x = 0.3, Br = 0.1$, $textSr = 3, \varepsilon = 0.3, \text{We} = 0.03$, and $E = 1$. (c) Concentration distribution ϕ_{con} versus y for $M = 3, \alpha = 0.2, \text{Sc} = 3, x = 0.3, \text{Br} = 0.1, \text{Sr} = 3, \varepsilon = 0.3, \text{We} = 0.03$, and $E = 1$. (d) Concentration distribution ϕ_{con} versus y for $M = 3, \theta = 2, \text{Sc} = 3, x = 0.1, \text{Br} = 0.1, \text{Sr} = 3, \varepsilon = 0.3, \alpha = 0.6$, and $E = 1$.

while it decreases when the Hartman number M increases.

Figure 2 explains the variation of different parameters on the heat transfer coefficient Z. Figures 2a – d show that the absolute value of heat transfer coefficient increases by increasing Br, ε, We, α, and E.

Figure 3 elucidates the concentration distribution of the fluid for the different parameters. The obtained results agree well with the observations in biological practice. It is obvious that transport of nutrients from blood takes place by the process of diffusion out of the blood vessels to the surrounding cells and tissues. A higher concentration at walls than along the axis is the general observation of the set of Figures 3a – d. Moreover these figures show that with an increase in Brinkman number Br, Schmidt number Sc, Weissenberg number We, and flow rate θ, the concentration field decreases.

6. Concluding Remarks

The effects of heat and mass transfer on the peristaltic motion of a magnetohydrodynamic Phan-Thien-Tanner fluid are analysed. The flow quantities of interest have been computed by regular perturbation method. Explicit attension is paid to temperature and concentration distribution. The main conclusions are summarized as follows: The temperature distribution increases with increasing values of $\mathrm{Br}, \varepsilon, \mathrm{We}$, and θ at the centre of the channel. The concentration distribution has an opposite behaviour to the temperature distribution for $\mathrm{Br}, \varepsilon, \mathrm{We}$, and θ.

[1] S. Srinivas and R. Gayathri, Appl. Math. Comput. **215**, 185 (2009).
[2] A. H. Shapiro, M. Y. Jaffrin, and S. L. Weinberg, J. Fluid Mech. **37**, 799 (1969).
[3] K. S. Mekheimer, Appl. Math. Comput. **153**, 763 (2004).
[4] M. Elshahed and M. H. Haroun, Math. Probl. Eng. **6**, 663 (2005).
[5] K. S. Mekheimer, Phys. Lett. A **372**, 4271 (2008).
[6] M. Kothandapani and S. Srinivas, Int. J. Nonlin. Mech. **43**, 915 (2008).
[7] K. S. Mekheimer and Y. Abd elmaboud, Physica A **387**, 2403 (2008).
[8] M. H. Haroun, Commun. Nonlin. Sci. Numer. Simul. **12**, 1464 (2007).
[9] Abd El Hakeem Abd El Naby, A. E. M. El Misery, and M. F. Abd El Kareem, Physica A **367**, 79 (2006).
[10] Abd El Hakeem Abd El Naby, J. Appl. Mech. **76**, 064504 (2009).
[11] S. Nadeem, T. Hayat, N. S. Akbar, and M. Y. Malik, Int. J. Heat Mass Trans. **52**, 4722 (2009).
[12] S. Noreen and M. Qasim, Eur. Phys. J. Plus **128**, 91 (2013).
[13] K. S. Mekheimer, J. Appl. Math. **2008**, 570825 (2008).
[14] Y. Abd elmaboud, K. S. Mekheimer, and A. I. Abdellateef, J. Heat Trans. **135** 044502 (2013).
[15] Y. Abd elmaboud and K. S. Mekheimer, Z. Naturforsch. **67a**, 185 (2012).
[16] K. S. Mekheimer and Y. Abd elmaboud, Phys. Lett. A **372**, 1657 (2008).
[17] K. Vajravelu, G. Radhakrishnamacharya, and V. Radhakrishnamurty, Int. J. Nonlin. Mech. **42**, 754 (2007).
[18] T. Hayat and N. Ali, Physica A **370**, 225 (2006).
[19] T. Hayat and N. Ali, Math. Comput. Model. **48**, 721 (2008).
[20] T. Hayat, M. U. Qureshi, and N. Ali, Phys. Lett. A **372**, 2653 (2008).
[21] T. Hayat, Y. Khan, N. Ali, and K. S. Mekheimer, Numer. Meth. Part. Diff. Eqs. **26**, 345 (2010).
[22] S. Srinivas and M. Kothandapani, Int. Commun. Heat Mass Trans. **35**, 514 (2008).
[23] A. Ogulu, Int. Commun. Heat Mass Trans. **33**, 790 (2006).
[24] N. Phan-Thien and R. I. Tanner, J. Non-Newtonian Fluid Mech. **2**, 353 (1977).
[25] N. Phan-Thien, J. Rheol. **22**, 259 (1978).