Elastic and Inelastic Interaction Behaviours for the $(2+1)$-Dimensional Nizhnik–Novikov–Veselov Equation in Water Waves

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A modified mapping method and new ansätz form are used to derive three families of variable separation solutions with two arbitrary functions of the $(2+1)$-dimensional Nizhnik–Novikov–Veselov equation in water waves. By selecting appropriate functions in the variable separation solution, we discuss interaction behaviours among dromion-pair and dromion-like peakon-pair and dromion-like semifoldon-pair. The analysis results exhibit that the interaction behaviours between dromion-pair and dromion-like peakon-pair, dromion-pair and semifoldon-pair, dromion-like peakon-pair and semifoldon-pair are all incomplete elastic, and there exists a phase shift. The interaction behaviour between two dromion-like semifoldon-pairs is completely elastic, and no phase shift appears after interaction. Moreover, during the interactions between dromion-pair and semifoldon-pair, dromion-like peakon-pair and semifoldon-pair, and between two dromion-like semifoldon-pairs, there all exist a multi-valued semifoldon-pair.

Key words: Modified Mapping Method; Nizhnik–Novikov–Veselov Equation; Elastic and Inelastic Interaction.

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1. Introduction

The dynamical behaviours of the finite amplitude waves on the free surface of an irrotational fluid has attracted tremendous attention over the last 40 years. Shallow water waves and a great deal of long wave phenomena are commonly studied by various models of nonlinear partial differential equations (PDEs). In linear wave theory, the Fourier analysis and the variable separation approach (VSA) are two most universal and powerful means to study the linear PDEs. In nonlinear domain, the counterparts (the celebrated inverse scattering method [1] and VSA [2 – 12]) have also developed and play an important role to analyze nonlinear wave dynamics. Many VSAs in nonlinear field have also been established, such as the multilinear VSA [2, 3] and the VSA based on mapping method [4 – 6], and so on. Moreover, many direct methods based on different mapping equations, which used to obtain travelling wave solutions, were extended to realize the variable separation of nonlinear PDEs, including the improved projective approach [7 – 9], the q-deformed hyperbolic functions method [10], and the projective Ricatti equation method (PREM) [11, 12].

Abundant localized coherent structures have been investigated based on various variable separation solutions [2 – 12]. Moreover, besides single-valued localized structures such as dromions, peakons, and compactons etc., many multi-valued structures including foldons and semi-foldons have also been a surge of interest due to their extensive applications in very complicated folded phenomena such as the folded protein [13], folded brain and skin surfaces, and many other kinds of folded biologic systems [14]. Many authors have also discussed interaction behaviours among these localized coherent structures. For example, the completely elastic interactions between dromions [2] and between dromion-solitoffs [8] have been reported. The incompletely elastic interactions between peakon and semifoldon [12] has been investigated. The completely inelastic interactions between peakon [3] and between semifoldons [10]. However, the interactions among some multi-soliton structures of dromions, peakons, and foldons were little reported in previous literature.

Naturally, some significant and interesting issues arise: Can other mapping equation be used to obtain variable separation solutions of some $(2+1)$-dimensional nonlinear physics systems? Based on these variable separation solutions, can we discuss some new dynamical behaviours among semi-structures? In order to answer these issues, we study the following well-known $(2+1)$-dimensional Nizhnik–Novikov–Veselov (NNV) equation

$$\begin{aligned} &u_t + au_{xxx} + bu_{yyy} - 3a(uv)_x - 3b(uw)_y = 0,\\ &u_x = v_y,\ \ u_y = w_x, \end{aligned} \tag{1}$$

where a and b are arbitrary constants. This system is simply a known isotropic Lax extension of the well-known $(1+1)$-dimensional shallow water-wave Korteweg–de Vries (KdV) model [15]. Some types of the soliton solutions have been studied by many authors. For instance, Boiti et al. [16] solved the NNV equation via the inverse scattering transformation. Tagami [17] obtained the soliton-like solutions of (1) by means of the Bäklund transformation. Ohta [18] obtained the Pfaffian solutions for (1). We also discussed some novel localized coherent structures about multi-valued functions [19, 20].

2. The Modified Mapping Method

Let us consider a given nonlinear PDE with independent variables $x = (x_0 = t, x_1, x_2, x_3, \ldots, x_m)$ and dependent variable u,

$$L(u, u_t, u_{x_i}, u_{x_i x_j}, \cdots) = 0, \tag{2}$$

where L is in general a polynomial function of its argument, and the subscripts denote the partial derivatives.

The basic idea of the mapping method is to seek for its ansätz

$$\begin{aligned} u = a_0(x) + \sum_{i=1}^{n} \Bigg\{ & a_i(x)\phi^i[q(x)] + \frac{b_i(x)}{\phi^i[q(x)]} \\ & + c_i(x)\phi^{i-1}[q(x)]\sqrt{\{A\phi[q(x)] - C\}\{B\phi[q(x)] - D\}} \Bigg\}, \end{aligned} \tag{3}$$

where a_i, b_i, c_i, and q are arbitrary functions of $\{x\}$ to be determined, n is fixed by balancing the linear term of the highest order with the nonlinear term in (2), and ϕ satisfies a mapping equation [4–9]. Here the superscript i indicates the power of ϕ, and A, B, C, and D are arbitrary constants.

Note that many mapping equations for ϕ have been used, such as the Riccati equation $\phi' = l_0 + \phi^2$ (l_0 is a constant and the prime denotes differentiation with respect to q) [4–6], $\phi' = \sigma\phi + \phi^2$ (σ is a constant) [7–9], and $\phi' = l_1 + l_2\phi^2$ (l_1 and l_2 are free constants) [19]. Here we seek for the solution of the given nonlinear PDE (2) with new mapping equation [21, 22]

$$\phi' = (A\phi - C)(B\phi - D), \tag{4}$$

which is known to possess the general solution

$$\phi = \frac{D\exp[(BC - AD)q] - C\exp[C_1(AD - BC)]}{B\exp[(BC - AD)q] - A\exp[C_1(AD - BC)]}. \tag{5}$$

Here C_1 is an integration constant.

To determined u explicitly, we take the following three steps:

Step 1: Determine n by balancing the highest nonlinear terms and the highest-order partial differential terms in the given nonlinear PDE (2).

Step 2: Substituting (3) along with (4) into (2) yields a set of polynomials for $\phi^i\sqrt{(A\phi - C)(B\phi - D)}$. Eliminating all the coefficients of the powers of $\phi^i\sqrt{(A\phi - C)(B\phi - D)}$ yields a series of partial differential equations, from which the parameters a_i, b_i, c_i, and q are explicitly determined.

Step 3: By substituting a_i, b_i, c_i, q, and (5) into (3), one can obtain possible solutions of (2).

3. Variable Separation Solutions for the $(2+1)$-Dimensional NNV Equation

Along with the modified mapping method in Section 2, by balancing the highest-order derivative terms with the nonlinear terms in (1), we suppose that it has the following formal solutions:

$$\begin{aligned} u = {}& \frac{a_{-2}}{\phi^2} + \frac{a_{-1}}{\phi} + a_0 + a_1\phi + a_2\phi^2 \\ & + a_3\sqrt{(A\phi - C)(B\phi - D)} + a_4\phi\sqrt{(A\phi - C)(B\phi - D)},\\ v = {}& \frac{b_{-2}}{\phi^2} + \frac{b_{-1}}{\phi} + b_0 + b_1\phi + b_2\phi^2 \\ & + b_3\sqrt{(A\phi - C)(B\phi - D)} + b_4\phi\sqrt{(A\phi - C)(B\phi - D)},\\ w = {}& \frac{c_{-2}}{\phi^2} + \frac{c_{-1}}{\phi} + c_0 + c_1\phi + c_2\phi^2 \\ & + c_3\sqrt{(A\phi - C)(B\phi - D)} + c_4\phi\sqrt{(A\phi - C)(B\phi - D)}, \end{aligned} \tag{6}$$

where a_i, b_i, c_i $(i = -2, -1, \ldots, 4)$ are all arbitrary functions of $\{x, y, t\}$, ϕ satisfies (5), and $q \equiv q(x, y, t)$. Inserting (6) into (1), selecting the variable separation ansatz

$$q = \chi(x,t) + \psi(y,t), \tag{7}$$

and eliminating all the coefficients of the powers of $\phi^i\sqrt{(A\phi - C)(B\phi - D)}$, one gets a set of PDEs, from which we have three kinds of solutions, namely

Solution 1

$$\begin{aligned}
&a_{-1} = a_{-2} = a_3 = a_4 = 0,\ a_0 = -2A^2D^2\chi_x\psi_y,\\
&a_1 = -2AB(AD+BC)\chi_x\psi_y,\ a_2 = 2A^2B^2\chi_x\psi_y,\\
&b_{-1} = b_{-2} = b_3 = b_4 = 0,\\
&b_0 = \frac{a\chi_{xxx} + \chi_t - 2aA^2D^2\chi_x^3}{3a\chi_x},\\
&b_1 = -2AB[(AD+BC)\chi_x^2 - \chi_{xx}],\\
&b_2 = 2A^2B^2\chi_x^2,\\
&c_{-1} = c_{-2} = c_3 = c_4 = 0,\\
&c_0 = \frac{b\psi_{yyy} + \psi_t - 2bA^2D^2\psi_y^3}{3b\psi_y},\\
&c_1 = -2AB[(AD+BC)\psi_y^2 - \psi_{yy}],\\
&c_2 = 2A^2B^2\psi_y^2,
\end{aligned} \tag{8}$$

Solution 2

$$\begin{aligned}
&a_{-1} = a_{-2} = a_1 = a_3 = 0,\\
&a_0 = -A^2D^2\chi_x\psi_y,\ a_2 = A^2B^2\chi_x\psi_y,\\
&a_4 = AB\sqrt{AB}\chi_x\psi_y,\ AD + BC = 0,\\
&b_{-1} = b_{-2} = 0,\ b_0 = \frac{a\chi_{xxx} + \chi_t - 2aA^2D^2\chi_x^3}{3a\chi_x},\\
&b_1 = AB\chi_{xx},\ b_2 = A^2B^2\chi_x^2,\\
&b_3 = \sqrt{AB}\chi_{xx},\ b_4 = AB\sqrt{AB}\chi_x^2,\\
&c_{-1} = c_{-2} = 0,\\
&c_0 = \frac{b\psi_{yyy} + \psi_t - 2bA^2D^2\psi_y^3}{3b\psi_y},\\
&c_1 = AB\psi_{yy},\ c_2 = A^2B^2\psi_y^2,\\
&c_3 = \sqrt{AB}\psi_{yy},\ c_4 = AB\sqrt{AB}\psi_y^2,
\end{aligned} \tag{9}$$

and Solution 3

$$\begin{aligned}
&a_{-2} = 2A^2D^4\chi_x\psi_y/B^2,\ a_0 = -4A^2D^2\chi_x\psi_y,\\
&a_2 = 2A^2B^2\chi_x\psi_y,\\
&b_{-1} = \frac{2AD^2\chi_{xx}}{B},\ b_{-2} = \frac{2A^2D^4\chi_x^2}{B^2},\\
&b_0 = \frac{a\chi_{xxx} + \chi_t + 4aA^2D^2\chi_x^3}{3a\chi_x},\\
&b_1 = 2AB\chi_{xx},\ b_2 = 2A^2B^2\chi_x^2,\\
&c_{-1} = \frac{2AD^2\psi_{yy}}{B},\ c_{-2} = \frac{2A^2D^4\psi_y^2}{B^2},\\
&c_0 = \frac{b\psi_{yyy} + \psi_t + 4bA^2D^2\psi_y^3}{3b\psi_y},\\
&c_1 = 2AB\psi_{yy},\ c_2 = 2A^2B^2\psi_y^2,\\
&a_{-1} = a_1 = a_3 = a_4 = b_3 = b_4 = c_3 = c_4 = 0,\\
&AD + BC = 0,
\end{aligned} \tag{10}$$

where χ and ψ are arbitrary functions of $\{x, t\}$ and $\{y, t\}$, respectively.

Therefore, the variable separation solution of the (2 + 1)-dimensional NNV equation reads

Family 1

$$\begin{aligned}
u = &-2A^2D^2\chi_x\psi_y - 2AB(AD+BC)\chi_x\psi_y\frac{\Theta}{\Lambda}\\
&+ 2A^2B^2\chi_x\psi_y\left(\frac{\Theta}{\Lambda}\right)^2,
\end{aligned} \tag{11}$$

$$\begin{aligned}
v = &\frac{a\chi_{xxx} + \chi_t - 2aA^2D^2\chi_x^3}{3a\chi_x}\\
&- 2AB[(AD+BC)\chi_x^2 - \chi_{xx}]\frac{\Theta}{\Lambda}\\
&+ 2A^2B^2\chi_x^2\left(\frac{\Theta}{\Lambda}\right)^2,
\end{aligned} \tag{12}$$

$$\begin{aligned}
w = &\frac{b\psi_{yyy} + \psi_t - 2bA^2D^2\psi_y^3}{3b\psi_y}\\
&- 2AB[(AD+BC)\psi_y^2 - \psi_{yy}]\frac{\Theta}{\Lambda}\\
&+ 2A^2B^2\psi_y^2\left(\frac{\Theta}{\Lambda}\right)^2,
\end{aligned} \tag{13}$$

Family 2

$$\begin{aligned}
u = &-A^2D^2\chi_x\psi_y + A^2D^2\chi_x\psi_y\frac{\Gamma_+}{\Gamma_-}\\
&\cdot\left[\frac{\Gamma_+}{\Gamma_-} + \sqrt{\left(\frac{\Gamma_+}{\Gamma_-} + 1\right)\left(\frac{\Gamma_+}{\Gamma_-} - 1\right)}\right],
\end{aligned} \tag{14}$$

$$v = \frac{a\chi_{xxx} + \chi_t - 2aA^2D^2\chi_x^3}{3a\chi_x} - AD\chi_{xx}\left[\frac{\Gamma_+}{\Gamma_-} - \sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}\right] + A^2D^2\chi_x^2\frac{\Gamma_+}{\Gamma_-} \cdot \left[\frac{\Gamma_+}{\Gamma_-} + \sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}\right], \quad (15)$$

$$w = \frac{b\psi_{yyy} + \psi_t + 4bA^2D^2\psi_y^3}{3b\psi_y} - AD\psi_{yy}\left[\frac{\Gamma_+}{\Gamma_-} - \sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}\right] + A^2D^2\psi_y^2\frac{\Gamma_+}{\Gamma_-} \cdot \left[\frac{\Gamma_+}{\Gamma_-} + \sqrt{\left(\frac{\Gamma_+}{\Gamma_-}+1\right)\left(\frac{\Gamma_+}{\Gamma_-}-1\right)}\right], \quad (16)$$

Family 3

$$u = -4A^2D^2\chi_x\psi_y + 2A^2D^2\chi_x\psi_y\left[\left(\frac{\Gamma_+}{\Gamma_-}\right)^2 + \left(\frac{\Gamma_-}{\Gamma_+}\right)^2\right], \quad (17)$$

$$v = \frac{a\chi_{xxx} + \chi_t + 4aA^2D^2\chi_x^3}{3a\chi_x} - 2AD\chi_{xx}\left[\frac{\Gamma_+}{\Gamma_-} + \frac{\Gamma_-}{\Gamma_+}\right] + 2A^2D^2\chi_x^2\left[\left(\frac{\Gamma_+}{\Gamma_-}\right)^2 + \left(\frac{\Gamma_-}{\Gamma_+}\right)^2\right], \quad (18)$$

$$w = \frac{b\psi_{yyy} + \psi_t + 4bA^2D^2\psi_y^3}{3b\psi_y} - 2AD\psi_{yy}\left[\frac{\Gamma_+}{\Gamma_-} + \frac{\Gamma_-}{\Gamma_+}\right] + 2A^2D^2\psi_y^2\left[\left(\frac{\Gamma_+}{\Gamma_-}\right)^2 + \left(\frac{\Gamma_-}{\Gamma_+}\right)^2\right], \quad (19)$$

where $\Theta = D\exp[(BC-AD)(\chi+\psi)] - C\exp[C_1(AD - BC)]$, $\Lambda = B\exp[(BC-AD)(\chi+\psi)] - A\exp[C_1(AD - BC)]$, $\Gamma_\pm = A\exp(2ADC_1) \pm B\exp[-2AD(\chi+\psi)]$.

4. Interaction Behaviours Among Special Soliton-Pairs

Based on the quantities u, v, and w expressed by (11) – (19), we can obtain many rich coherent localized structures such as nonpropagating solitons, dromions, peakons, compactons, foldons, instantons, and ring solitons discussed in [2 – 12]. Here we omit them, and pay attention to interaction behaviours between special soliton-pairs for the physical quantity u expressed by (14).

4.1. Localized Structures Constructed by Multi-Valued Functions

We discuss the three special combined soliton-pair structures, i.e. dromion-pair and dromion-like peakon-pair and dromion-like semifoldon-pair by introducing multi-valued function as

$$\chi_x = \sum_{i=1}^{N} \kappa_i(\zeta - d_i t), \quad x = \zeta + \sum_{i=1}^{N} \eta_i(\zeta - d_i t), \quad (20)$$

where d_i $(i = 1, 2, \ldots, N)$ are arbitrary constants, κ_i and η_i are localized excitations with the properties $\kappa_i(\pm\infty) = 0$, $\eta_i(\pm\infty) = \text{consts}$. From (20), one can know that ζ may be a multi-valued function in some suitable regions of x by choosing the functions η_i appropriately. Therefore, the function p_x, which is obviously an interaction solution of N localized excitations due to the property $\zeta|_{x\to\infty} \to \infty$, may be a multi-valued function of x in these areas, though it is a single-valued function of ζ. Actually, most of the known multi-loop solutions are special cases of (20).

Specifically, χ and ψ are chosen as

$$\chi_x = 0.5\,\text{sech}^2(\zeta - 0.5t), \quad x = \zeta - E\tanh(\zeta - 0.5t), \quad (21)$$

$$\psi = \tanh(0.5y - 5) - 0.55\tanh(0.5y + 5), \quad (22)$$

where E is a characteristic parameter, which determines the localized structure. Figure 1 describes these special localized structures, i.e. special dromion-pair (a dipole type dromion with one up and one down bounded peaks), dromion-like peakon-pair, dromion-like semifoldon-pair with $E = 0.2, 1, 1.5$, respectively. They localize as bell-pair in the y-direction and bell-like soliton, peakon, and loop soliton in the x-direction, respectively.

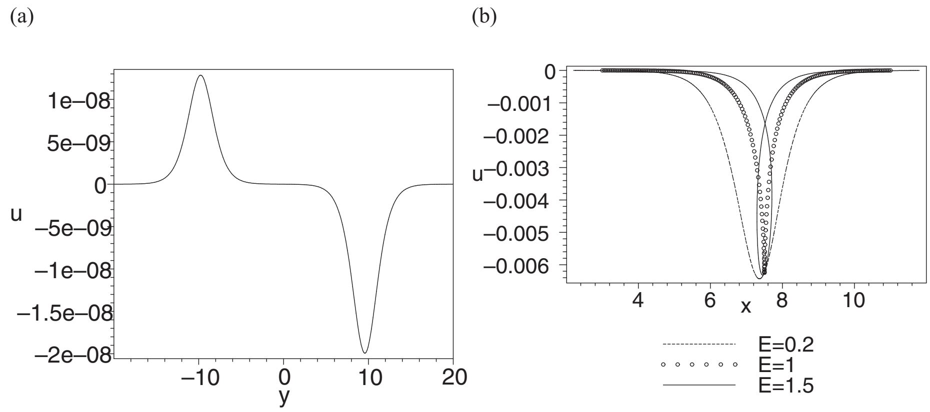

Fig. 1. Sectional views of special soliton-pair at (a) $x = 0$ and (b) $y = 8$ for parameters $A = C_1 = 1, B = 2, C = 0.25, D = -0.5$ at time $t = 15$.

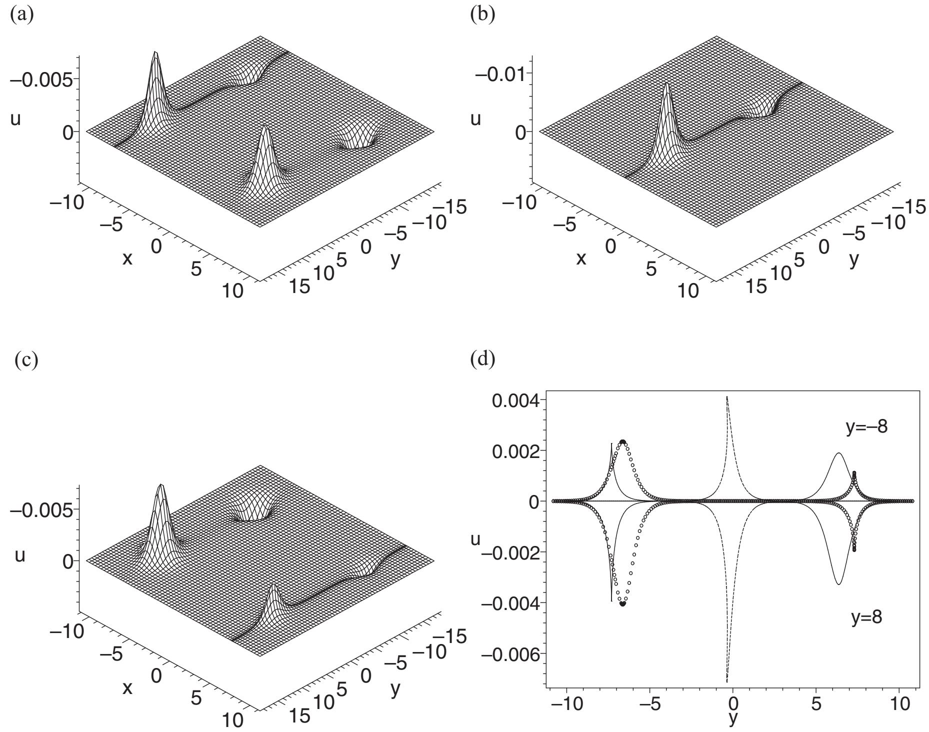

Fig. 2. Incompletely elastic interaction between special dromion-pair and dromion-like peakon-pair at time (a) $t = -15$, (b) $t = -1$, and (c) $t = 15$. (d) Sectional views of (a) – (c) at $y = -8, 8$ when $t = -15$ (solid line), $t = -1$ (dash line), and $t = 15$ (circle). The parameters are chosen as $A = C_1 = 1, B = 2, C = 0.25, D = -0.5, E = 1, F = 0.2$.

4.2. Incompletely Elastic Interaction Among Solitons

Let us study interaction behaviours among these special solitons produced by the multi-valued functions above. If we take the specific choice $N = 2, d_1 = 0.5$, and $d_2 = -0.5$ in (20), one has

$$\chi_x = 0.7\mathrm{sech}^2(\zeta - 0.5t) + 0.9\mathrm{sech}^2(\zeta + 0.5t),$$

$$x = \zeta - E\tanh(\zeta - 0.5t) - F\tanh(\zeta + 0.5t), \quad (23)$$

where E and F are characteristic parameters, which determine the types of interaction. Further, ψ is given by (21). From the expression u (14), one can obtain two solitons, one is moving along the positive x-direction and another is moving along the negative x-direction.

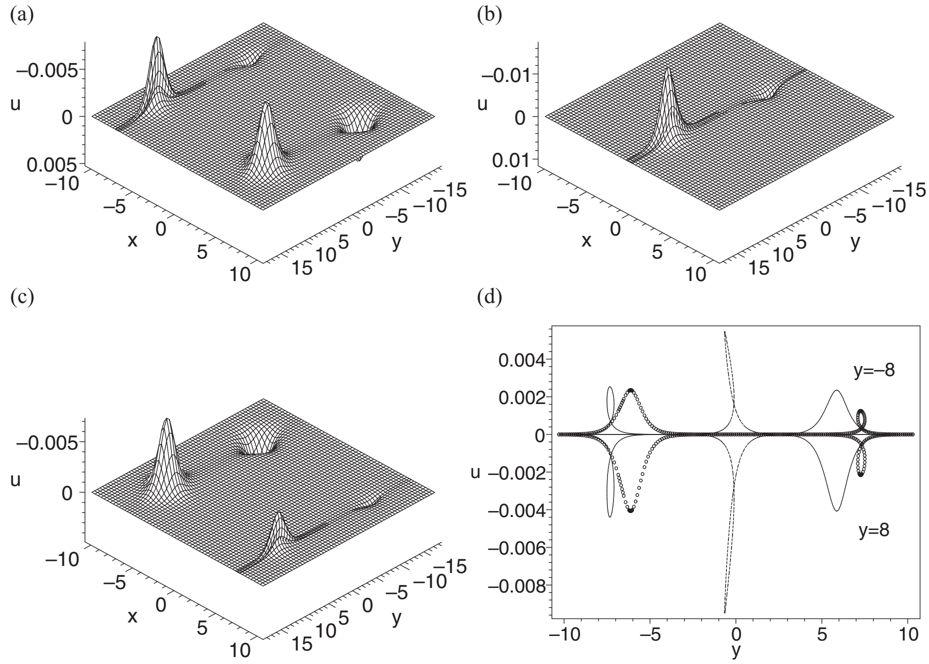

Fig. 3. Incompletely elastic interaction between dromion-pair and semifoldon-pair at time (a) $t = -15$, (b) $t = -1$, and (c) $t = 15$. (d) Sectional views of (a) – (c) at $y = -8, 8$ when $t = -15$ (solid line), $t = -1$ (dash line), and $t = 15$ (circle). The parameters are chosen as $A = C_1 = 1, B = 2, C = 0.25, D = -0.5, E = 1.5, F = 0.2$.

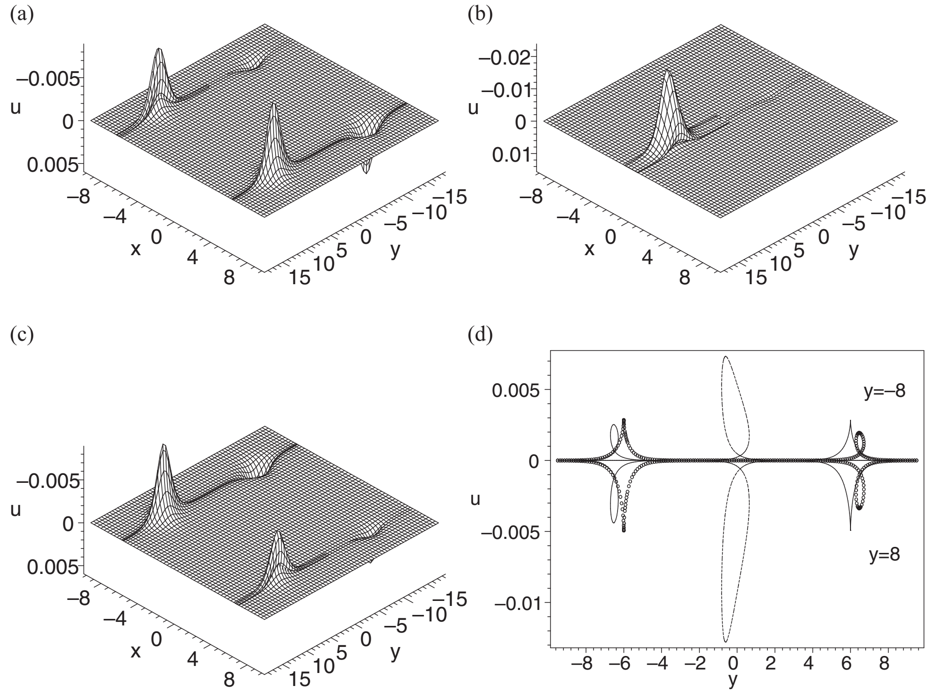

Fig. 4. Incompletely elastic interaction between peakon-pair and semifoldon-pair at time (a) $t = -15$, (b) $t = -1$, and (c) $t = 15$. (d) Sectional views of (a) – (c) at $y = -8, 8$ when $t = -15$ (solid line), $t = -1$ (dash line), and $t = 15$ (circle). The parameters are chosen as $A = C_1 = 1, B = 2, C = 0.25, D = -0.5, E = 1.5, F = 1$.

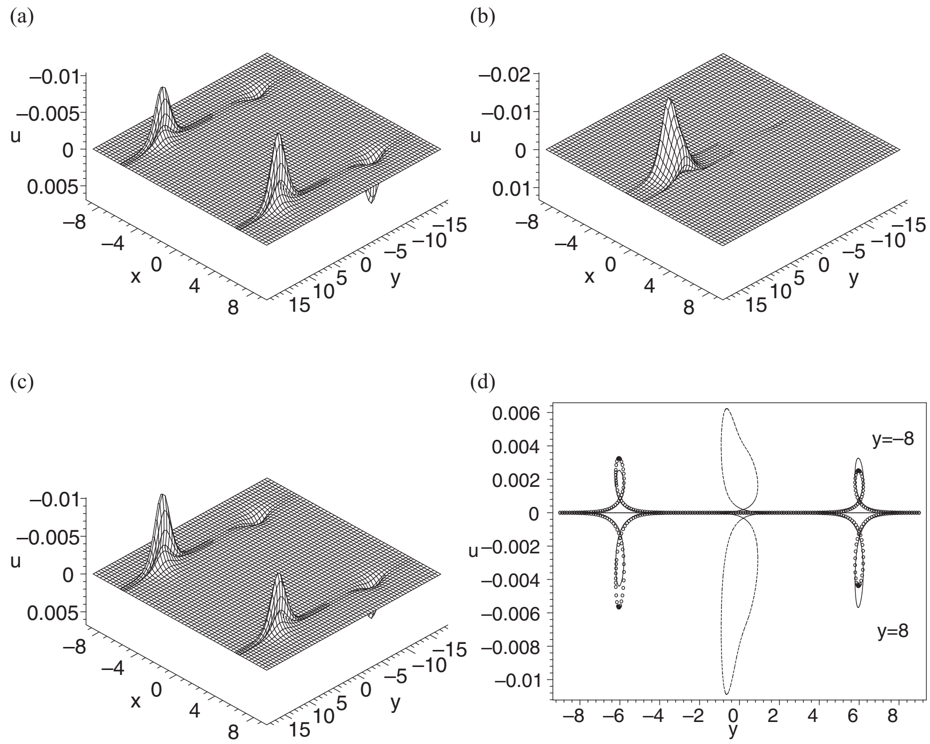

Fig. 5. Completely elastic interaction between semifoldon-pairs at time (a) $t=-15$, (b) $t=-1$, and (c) $t=15$. (d) Sectional views of (a) – (c) at $y=-8,8$ when $t=-15$ (solid line), $t=-1$ (dash line), and $t=15$ (circle). The parameters are chosen as $A=C_1=1, B=2, C=0.25, D=-0.5, E=F=1.5$.

The interactions between solitons may be regarded as elastic or inelastic. It is called completely elastic, if the amplitude, velocity, and wave shape of solitons do not changed after their interaction. Otherwise, the interactions between solitons are inelastic (incompletely elastic and completely inelastic). Like the collisions between two classical particles, a collision in which solitons stick together is sometimes called completely inelastic.

If we take the specific values $E=1, F=0.2$ in (23), then we can successfully construct the interaction between a dromion-like peakon-pair and a special dromion-pair, of which possess a phase shift for the physical quantity u depicted in Figure 2. From Figure 2, one can find that the interaction may exhibit a incompletely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction, and there exists a peakon-pair (dash-line in Fig. 2d) in the process of their collision. The phase shift can be observed. Prior to interaction, the velocities of the smaller dromion-pair and the lager dromion-like peakon-pair have set to be $\{v_{01x}=d_1=0.5\}$ and $\{v_{02x}=d_2=-0.5\}$, respectively. The final velocities v_{1x} and v_{2x} of the moving solitons also completely maintain their initial velocities $\{v_{1x}=v_{01x}=0.5\}$ and $\{v_{2x}=v_{02x}=-0.5\}$. However, two solitons do not exchange the corresponding positions and shift some distances.

In the following, we discuss the interaction between a dromion-like semifoldon-pair and a special dromion-pair for the specific values $E=1.5, F=0.2$ in (23). This interaction is also a incompletely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction (c.f. Fig. 3). After interaction, the dromion-pair and the dromion-like semifoldon-pair maintain their initial velocities $\{v_{1x}=v_{01x}=0.5\}$ and $\{v_{2x}=v_{02x}=-0.5\}$, respectively. From Figure 3d we learn that the two solitons do not exchange the corresponding positions and shift some distances. Note that there exists a multi-valued semifoldon-pair (dash-line in Fig. 3d) in the process of their collision, which is different from the case of the interaction between dromion-like peakon-pair and special dromion-pair.

Besides the two kind of interactions above, we can also investigate the interaction between a dromion-like semifoldon-pair and a dromion-like peakon-pair by choosing parameters $E=1.5, F=1$ in (23). This interaction shows also an incompletely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction, and there exists a phase shift because the two solitons do not exchange the corresponding positions. In the process of their collision, a multi-valued semifoldon-pair (dash-line in Fig. 4d) also appears, and the two solitons also preserve their initial velocities after interaction.

4.3. Completely Elastic Interaction Among Solitons

It is interesting to note that although the above selections are all incompletely elastic interaction behaviours, we can also construct localized coherent structures with completely elastic interaction behaviours by appropriately selecting the values of E and F in (23).

If we select the specific values $E=F=1.5$ in (23), then we can successfully construct the interaction between two dromion-like semifoldon-pairs for the physical quantity u depicted in Figure 5. From Figure 5, one can find that the interaction among them may exhibit a completely elastic behaviour since solitons' shapes, amplitudes, and velocities are completely maintained after interaction. The phase shift is not observed, and two solitons exchange the corresponding positions. Similar to the two kinds of interactions in Figures 3 and 4, there exists a multi-valued semifoldon-pair (dash-line in Fig. Figure 5d) during their collision.

5. Summary and Discussion

In this paper, we obtained three families of variable separation solutions with two arbitrary functions of the $(2+1)$-dimensional Nizhnik–Novikov–Veselov equation in water waves, and discussed interaction behaviours among some special soliton-pairs. The main points are as follows:

• A new mapping equation and new ansätz form are used.

Besides mapping equations in [4–9, 19], a new mapping equation was utilized to obtain variable separation solutions of some $(2+1)$-dimensional nonlinear physics systems. As an example, we applied it to the $(2+1)$-dimensional NNV equation, and derived three families of variable separation solutions with two arbitrary functions. Moreover, the ansätz form (3) is more general than those in [4–9, 19].

• Elastic interactions of special solitons are investigated.

By selecting appropriate functions in the variable separation solution, we discussed interaction behaviours among special solitons, constructed by multi-valued functions, including the dromion-pair and dromion-like peakon-pair and dromion-like semifoldon-pair. The analysis results exhibit that the interaction behaviours between dromion-pair and dromion-like peakon-pair, dromion-pair and semifoldon-pair, dromion-like peakon-pair and semifoldon-pair are all incomplete elastic, and there exists a phase shift. The interaction behaviour between two dromion-like semifoldon-pairs is completely elastic, and no phase shift appears after interaction. Moreover, during the interactions between dromion-pair and semifoldon-pair, dromion-like peakon-pair and semifoldon-pair, and between two dromion-like semifoldon-pairs, there all exists a multi-valued semifoldon-pair.

Of course, the method presented in this paper can be further extended to $(1+1)$-dimensional and $(3+1)$-dimensional nonlinear systems.

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