Influence of Electron Exchange and Quantum Shielding on the Elastic Collisions in Quantum Plasmas

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The influence of electron exchange and quantum shielding on the elastic electron–ion collision is investigated in degenerate quantum plasmas. The second-order eikonal method and effective screened potential are employed to obtain the scattering phase shift and collision cross section as functions of the impact parameter, collision energy, electron-exchange parameter, Fermi energy, and plasmon energy. It is found that the electron-exchange effect enhances the eikonal scattering phase shift as well as the eikonal collision cross section in quantum plasmas. The maximum position of the differential eikonal collision cross section is found to be receded from the collision center with an increase of the electron-exchange effect. It is interesting to note that the influence of the electron exchange on the eikonal collision cross section decreases with increasing collision energy. It is also found that the eikonal collision cross section decreases with an increase of the plasmon energy and, however, increases with increasing Fermi energy.

Key words: Eikonal Cross Section; Electron-Exchange Effect; Quantum Plasmas.

1. Introduction

The atomic processes in plasmas have received considerable attention since these processes have been extensively used as plasma diagnostics to provide useful information on various plasma parameters [1 – 7]. Especially, the elastic electron–ion collision has been known as one of the most fundamental atomic collision processes in plasmas and also has provided useful information on the collision system as well as the characteristics of surrounding plasmas. Recently, there has been a great interest in the investigation of the physical properties of various quantum plasmas since the quantum plasmas have been found in many nanoscale objects in modern science and technology such as nanowires, quantum dot, and semiconductor devices as well as astrophysical plasmas in compact objects [8 – 19]. It has been shown that the effective interaction potential in weakly coupled classical plasmas can be represented by the Debye–Hückel model [4, 20] since the average interaction energy between plasma particles is usually smaller than the average kinetic energy of a particle. However, it would be expected that the screened interaction potential in quantum plasmas would be quite different from the standard Debye–Hückel potential due to the influence of the Bohm potential and quantum statistical pressure caused by the quantum-mechanical and multiparticle correlation effects in dense quantum plasmas [14, 15]. In addition to the Bohm potential and quantum statistical pressure effects, it has been shown by Shukla and Eliasson [19] that the electron-exchange effect due to the electron-$1/2$ spin plays a crucial role in the formation of the electric potential and plasma dielectric function in quantum plasmas. However, the influence of the electron-exchange and quantum shielding on the elastic collision in quantum plasmas has not been investigated yet. Thus, in this paper, we investigate the electron-exchange and quantum shielding effects on the elastic electron–ion collision in degenerate quantum plasmas. The second-order eikonal analysis [21] and Shukla and Eliasson effective screened potential with the impact parameter method are employed to obtain the eikonal scattering phase shift and eikonal collision cross section as functions of the impact parameter, collision energy, electron-exchange parameter, Fermi energy, and plasmon energy. The variation of the electron-exchange and quantum screening effects on the eikonal scattering phase shift and eikonal collision cross section in quantum plasmas is also discussed.

2. Eikonal Phase and Cross Section

For a given interaction potential $V(r)$ in the electron–ion collision system, the semiclassical eikonal wave function $\psi_E(r) = N(r)\exp[iS(r)/\hbar]$ for the nonrelativistic Schrödinger equation would be obtained by the Hamilton–Jacobi equation [22, 23]

$$H(\nabla S(r), r) = \frac{1}{2\mu}[\nabla S(r)]^2 + V(r) = E, \qquad (1)$$

where $N(r)$ is the normalization factor, $S(r)$ the action function, and $H(\nabla S(r), r)$ the Hamiltonian. μ is the reduced mass of the collision system. It is E $(= \hbar^2 k^2/2\mu) = \mu v^2/2$ the collision energy with $k, \hbar$, and v the wave number, the rationalized Planck constant, and the collision velocity, respectively. In cylindrical coordinates with the straight-line trajectory, the solution of the Hamilton–Jacobi equation for the action function $S(r)$ would be represented by

$$S(r)/\hbar \cong k_i z - \frac{\mu}{\hbar^2 k_i}\int_{-\infty}^{z} dz' V(z', b), \qquad (2)$$

where $r = z\hat{n} + b$. $\hat{n}$ is the unit vector normal to the momentum transfer q $(\equiv k_i - k_f)$, k_i and k_f are, respectively, the incident and final wave vectors, and b is the impact parameter. It has been known that the validity of the semiclassical eikonal method is known as $|V(R)|/E < 1$ [21], where V is the typical strength of the interaction potential, and R is the interaction range. The semiclassical eikonal wave function $\psi_E(r)$ would be then represented by

$$\psi_E(r) \cong (2\pi)^{-\frac{3}{2}} \cdot \exp\left[ik_i z - i\frac{\mu}{\hbar^2 k_i}\int_{-\infty}^{z} dz' V(z', b)\right]. \qquad (3)$$

Hence, the corresponding semiclassical eikonal scattering amplitude $f_E(q)$ is obtained by the following integral representation:

$$f_E(q) = -\frac{\mu}{2\pi\hbar^2}\int d^3 r \cdot \exp\left[iq\cdot r - i\frac{\mu}{\hbar^2 k_i}\int_{-\infty}^{z} dz' V(z', b)\right] V(r). \qquad (4)$$

Since the differential eikonal collision cross section is determined by the relation $d\sigma_E/d\Omega = \left|f_E(q)\right|^2$, the total elastic eikonal collision cross section σ_E would be then expressed by

$$\sigma_E(k) = \int d^2 b \left|\exp\left[i\left(\chi_1(b)/k + \chi_2(b)/k^3\right)\right] - 1\right|^2 = 2\pi\int db b\left|\exp\left[i\chi_E(b,k)\right] - 1\right|^2, \qquad (5)$$

where $d\Omega$ is the differential solid angle, k $(\equiv |k_i| = |k_f|)$ is the elastic scattering wave number, $\chi_1(b)$ and $\chi_2(b)$ are, respectively, the first- and second-order eikonal scattering phase shifts [21]

$$\chi_1(b) = -\frac{\mu}{\hbar^2}\int_{-\infty}^{\infty} dz V(z, b), \qquad (6)$$

$$\chi_2(b) = \frac{\mu^2}{2\hbar^4}\int_{-\infty}^{\infty} dz \nabla\left[\int_{-\infty}^{z} dz' V(z', b)\right] \cdot \nabla\left[\int_{z}^{\infty} dz' V(z', b)\right]. \qquad (7)$$

Then, the total eikonal scattering phase $\chi_E(b,k)$ using the second-order eikonal method is found to be

$$\chi_E(b,k) = -\frac{\mu}{\hbar^2 k}\int_{-\infty}^{\infty} dz V(r) - \frac{\mu^2}{4\hbar^4 k^3}\int_{-\infty}^{\infty} dz\left[V(r) + r\frac{d}{dr}V(r)\right]V(r). \qquad (8)$$

In the nonrelativistic quantum hydrodynamic model [19], the continuity and momentum equations for degenerate quantum plasmas are, respectively, represented by

$$\frac{\partial n}{\partial t} + \nabla\cdot(nv) = 0, \qquad (9)$$

$$\frac{\partial v}{\partial t} + v\cdot\nabla v = -\frac{1}{mn}\nabla P + \frac{e}{m}\nabla\varphi + \frac{\hbar^2}{2m^2}\nabla\left(\frac{\nabla^2\sqrt{n}}{\sqrt{n}}\right) + \frac{1}{m}\nabla V_{XC}, \qquad (10)$$

where n is the number density of the electron, v, m, and P are, respectively, the velocity, the mass, and the pressure of the electron. φ is the electric potential, and

V_{XC} is the electron-exchange potential. In the Shukla–Eliasson model [19], the electron-exchange potential V_{XC} would be represented by $V_{XC} = -0.985e^2 n^{1/3}[1 + (0.034/a_0 n^{1/3})\ln(1+18.37a_0 n^{1/3})]$, where a_0 $(= \hbar^2/me^2)$ is the Bohr radius of the hydrogen atom. It has been known that the quantum hydrodynamic model would be quite useful to investigate the transport process in nanoscience. In (10), the pressure term is known as the quantum statistical effect due to the fermionic behaviour of the plasma electrons, the Laplacian operator term represents the Bohm potential effect due to the quantum-diffraction effect, and the V_{XC} term stands for the additional potential due to the electron-exchange effect caused by the electron spin. Very recently, Shukla and Eliasson [19] have obtained an extremely useful expression of the plasma dielectric function ε_{SE} in degenerate quantum plasmas including the influence of the electron exchange and quantum shielding with quasistationary density perturbations such as $\varepsilon_{SE}^{-1} = 1 + [(k^2/k_s^2) + \alpha k^4/k_s^4]/[1 + (k^2/k_s^2) + \alpha k^4/k_s^4]$, where k_s $[= \omega_p/(v_F^2/3 + v_{ex}^2)^{1/2}]$ represents the inverse effective Thomas–Fermi screening length, ω_p is the electron plasma frequency, v_F is the electron Fermi velocity, v_{ex} is the electron-exchange velocity associated with the electron-exchange effect, and α $\left[= \hbar\omega_p^2/4m^2(v_F^2/3 + v_{ex}^2)^2\right]$ is the quantum recoil parameter. It is also found that the effective electric potential $\varphi_{SE}(r)$ of a charge Q in quantum plasmas is obtained by $\varphi_{SE}(r) = (Q/2\pi^2)\int d^3k\, e^{ik\cdot r}/k^2\varepsilon_{SE}$ using the Shukla and Eliasson plasma dielectric function ε_{SE} when the plasmon energy E_p $(= \hbar\omega_p)$ is comparable or smaller than the Fermi energy E_F $(= mv_F^2/2)$ [19]. Using the effective electric potential model [19], the Shukla and Eliasson effective interaction potential $V_{SE}(r)$ between the projectile electron and target ion with nuclear charge Ze in degenerate quantum plasmas becomes

$$V_{SE}(r) = -\frac{Ze^2}{2r}\Big[(1+\xi)\exp(-k_+ r) + (1-\xi)\exp(-k_- r)\Big], \tag{11}$$

where $\xi \equiv (1-4\alpha)^{-1/2}$ and the effective inverse screening lengths $k_\pm$ are given by $k_\pm \equiv k_s[1 \mp (1-4\alpha)^{1/2}]^{1/2}/(2\alpha)^{1/2}$. It can be shown that, in the limit $\alpha \to 0$, the Shukla and Eliasson effective interaction potential $V_{SE}(r)$ would be the modified Thomas–Fermi screened Coulomb potential, i.e., $V_{TF}(r) = -(Ze^2/r)e^{-k_s r}$ since $k_+ \to k_s$ and $k_- \to \infty$ as $\alpha \to 0$. In dense semiclassical plasmas [17], the number density n and temperature T are known to be about $10^{20}-10^{24}$ cm^{-3} and $5\cdot 10^4 - 10^6$ K. In addition, it has been known that the physical properties of the dense semiclassical plasma [17] would be expressed by the plasma coupling parameter Γ $[= (Ze)^2/ak_BT]$, degeneracy parameter θ $(= k_BT/E_F)$, density parameter r_s $(= a/a_0)$, where a is the average distance between plasma particles. After some mathematical manipulations using the Shukla and Eliasson effective interaction potential $V_{SE}(r)$ and the impact parameter analysis with the identity of the jth-order modified Bessel function of the second kind, $K_j(\eta) = [\pi^{1/2}/(j-1/2)!](\eta/2)^j\int_1^\infty dt\, e^{-\eta t}(t^2-1)^{j-1/2}$, the total eikonal scattering phase shift χ_E $(\bar b, \bar E, \bar E_F, \bar E_p, \beta)$ obtained by the second-order eikonal method is found to be

$$\begin{aligned}
\chi_E(\bar b, \bar E, \bar E_F, \bar E_p, \beta) &= \frac{1}{\bar E^{1/2}}\Big\{\Big[1+\xi(\bar E_F, \bar E_p, \beta)\Big] \\
&\cdot K_0\Big[\bar k_+(\bar E_F, \bar E_p, \beta)\bar b\Big] + \Big[1-\xi(\bar E_F, \bar E_p, \beta)\Big] \\
&\cdot K_0\Big[\bar k_-(\bar E_F, \bar E_p, \beta)\bar b\Big]\Big\} + \frac{1}{8\bar E^{3/2}}\Big\{\Big[1+\xi(\bar E_F, \bar E_p,)\Big]^2 \\
&\cdot \bar k_+\Big(\bar E_F, \bar E_p, \beta\Big)K_0\Big[2\bar k_+(\bar E_F, \bar E_p, \beta)\bar b\Big] \\
&+ \Big[1-\xi^2(\bar E_F, \bar E_p, \beta)\Big]\Big[\bar k_+(\bar E_F, \bar E_p, \beta) \\
&+ \bar k_-(\bar E_F, \bar E_p, \beta)\Big] \times K_0\Big[\bar k_+(\bar E_F, \bar E_p, \beta)\bar b \\
&+ \bar k_-(\bar E_F, \bar E_p, \beta)\bar b\Big] + \Big[1-\xi(\bar E_F, \bar E_p, \beta)\Big]^2 \\
&\cdot \bar k_-(\bar E_F, \bar E_p, \beta)K_0\Big[2\bar k_-(\bar E_F, \bar E_p, \beta)\bar b\Big]\Big\},
\end{aligned} \tag{12}$$

where $\bar b$ $(\equiv b/a_Z)$ is the scaled impact parameter, a_Z $(= a_0/Z)$ is the Bohr radius of the hydrogenic ion with nuclear charge Ze, $\bar E$ $(\equiv \mu v^2/2Z^2\text{Ry})$ is the scaled collision energy, Ry $(= me^4/2\hbar^2 \approx 13.6$ eV$)$ is the Rydberg constant, $\bar E_F$ $(\equiv E_F/Z^2\text{Ry})$ is the scaled Fermi energy, $\bar E_p$ $(\equiv E_p/Z^2\text{Ry})$ is the scaled plasmon energy, β $(\equiv v_{ex}/v_F)$ stands for the electron-exchange parameter, $\xi(\bar E_F, \bar E_p, \beta) = [1-4\alpha(\bar E_F, \bar E_p, \beta)]^{-1/2}$, the quantum recoil parameter is represented by $\alpha(\bar E_F, \bar E_p, \beta) = (3\bar E_p/4\bar E_F)^2(1+3\beta^2)^{-2}$, the scaled shielding parameters $\bar k_\pm(\bar E_F, \bar E_p, \beta)$ in degenerate quantum plasmas are given by $\bar k_\pm(\bar E_F, \bar E_p, \beta)(\equiv \bar k_\pm a_Z) = \bar k_s(\bar E_F, \bar E_p, \beta)\Big\{1 \mp [1-4\alpha(\bar E_F, \bar E_p, \beta)]^{1/2}\Big\}^{1/2}/[2\alpha(\bar E_F, \bar E_p, \beta)]^{1/2}$, and $\bar k_s(\bar E_F, \bar E_p, \beta)(\equiv \bar k_s a_Z) = [(3\bar E_p^2/4\bar E_F)/(1+3\beta^2)]^{1/2}$. If

$\alpha \to 0$, i. e., the case of the Thomas–Fermi screened Coulomb interaction, i. e., $V_{\rm TF}(r) = -(Ze^2/r)\,{\rm e}^{-k_{\rm s}r}$, the total eikonal phase shift is obtained by $\chi'_{\rm E} = (2/\bar{E}^{1/2})K_0(\bar{k}_{\rm s}\bar{b}) + (1/2\bar{E}^{3/2})\bar{k}_{\rm s}K_0(2\bar{k}_{\rm s}\bar{b})$ since the shielding distance is determined by $k_{\rm s}^{-1}$. Hence, the scaled differential eikonal collision cross section $\partial\bar{\sigma}_{\rm E}[\equiv(\mathrm{d}\sigma_{\rm E}/\mathrm{d}\bar{b})/\pi a_Z^2]$ in units of πa_Z^2 for the elastic electron–ion collision including the electron exchange and quantum shielding is obtained by

$$\begin{aligned}\partial\bar{\sigma}_{\rm E}(\bar{b},\bar{E},\bar{E}_{\rm F},\bar{E}_{\rm p},\beta) &= 2\bar{b}\Bigg|\exp\Bigg\{\frac{\rm i}{\bar{E}^{1/2}}\\ &\Big[[1+\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]K_0[\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\\ &+[1-\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]K_0[\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\Big]\\ &+\frac{\rm i}{8\bar{E}^{3/2}}\Big[[1+\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]^2\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\\ &\cdot K_0[2\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]+[1-\xi^2(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]\\ &\cdot[\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)+\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]\\ &\cdot K_0[\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}+\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\\ &+[1-\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]^2\,\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\\ &\cdot K_0[2\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\Big]\Bigg\}-1\Bigg|^2.\end{aligned} \tag{13}$$

As shown in (13), the electron-exchange effect on the electron–ion collision process in quantum plasmas has been explicitly included through the parameter β in the effective shielding parameters $\bar{k}_\pm$ as well as the quantum recoil parameter α. For the Thomas–Fermi screened Coulomb interaction case $V_{\rm TF}(r) = -(Ze^2/r)\,{\rm e}^{-k_{\rm s}r}$, i. e., $\alpha \to 0$, the scaled differential eikonal collision cross section is then found to be $\partial\bar{\sigma}'_{\rm E} = 2\bar{b}|\exp[(2{\rm i}/\bar{E}^{1/2})K_0(\bar{k}_{\rm s}\bar{b}) + ({\rm i}/2\bar{E}^{3/2})\bar{k}_{\rm s}K_0(2\bar{k}_{\rm s}\bar{b})] - 1|^2$. Hence, the scaled total eikonal cross section $\bar{\sigma}_{\rm E}$ $(\equiv \sigma_{\rm E}/\pi a_Z^2)$ in units of πa_Z^2 for the elastic electron–ion collision in degenerate quantum plasmas including the influence of the electron exchange and quantum shielding is obtained by the following integral form:

$$\begin{aligned}\bar{\sigma}_{\rm E}(\bar{E},\bar{E}_{\rm F},\bar{E}_{\rm p},\beta) &= 2\int_0^{[(1+3\beta^2)/(3\bar{E}_{\rm p}^2/4\bar{E}_{\rm F})]^{1/2}}\\ &\cdot\mathrm{d}\bar{b}\bar{b}\Bigg|\exp\Bigg\{\frac{\rm i}{\bar{E}^{1/2}}\Big[[1+\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]\\ &\cdot K_0[\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]+[1-\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]\\ &\cdot K_0[\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\Big]+\frac{\rm i}{8\bar{E}^{3/2}}\Big[[1+\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]^2\\ &\cdot\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)K_0[2\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\\ &+[1-\xi^2(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)][\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\\ &+\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]\\ &\cdot K_0[\bar{k}_+(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}+\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\\ &+[1-\xi(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)]^2\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\\ &\cdot K_0[2\bar{k}_-(\bar{E}_{\rm F},\bar{E}_{\rm p},\beta)\bar{b}]\Big]\Bigg\}-1\Bigg|^2,\end{aligned} \tag{14}$$

where the upper limit of the integration is given by $[(1+3\beta^2)/(3\bar{E}_{\rm p}^2/4\bar{E}_{\rm F})]^{1/2}$ since the effective shielding length in quantum plasmas can be determined by the Fermi wave length. In order to explicitly investigate the electron exchange and quantum shielding effects on the elastic electron–ion collision process in degenerate quantum plasmas, we consider the energy domain $\bar{E} > 1$ since the second-order eikonal method is known to be valid for high-energy projectiles such as $v > Z\alpha_f$c [21], where α_f $(= e^2/\hbar{\rm c} \approx 1/137)$ is the fine structure constant and c is the speed of light. Recent years, several excellent investigations have provided the extremely useful effective interaction potentials to obtain electron–atom, electron––ion, and ion–atom interactions in dense semiclassical plasmas taking into accounts the symmetry and plasma degeneracy effects as well as quantum-mechanical and plasma screening effects [24–26]. However, the symmetry effect on the elastic electron–ion collision has not been considered in this work since the investigation of the electron exchange and quantum shielding on the elastic collision process is the main purpose of this work. The general thermodynamic Green's function approach for the scattering phase shifts can be found in an excellent work of Schmidt and Röpke [27]. In addition, a recent excellent work has provided useful information on the scattering phase shifts for the electron–atom interaction using the cluster-virial expansion with the Beth–Uhlenbeck approach [26]. It has been shown that one of the most important effects in quantum plasmas is the Pauli blocking effect. The influence of the Pauli blocking is not considered in this work since the investigation of the electron-exchange effect on the elastic electron–ion collision in degenerate quantum plasmas is the main purpose of this work. However, the investigation of the influence of the Pauli blocking on the elastic electron–ion collision in degenerate quantum plasmas will be treated elsewhere by using the generalized

Beth–Uhlenbeck formula described in a recent excellent work of Omarbakiyeva, Fortmann, Ramazanov, and Röpke [26].

3. Results and Discussions

Figure 1 represents the total eikonal scattering phase shift χ_{E} for the elastic electron–ion collision in quantum plasmas including the influence of the electron exchange and quantum shielding as a function of the scaled impact parameter $\bar{b}$ for various values of the electron-exchange parameter β. From this figure, it is shown that the eikonal scattering phase shift χ_{E} decreases with an increase of the impact parameter $\bar{b}$ and increases with increasing electron-exchange parameter β. Hence, we have found that the electron-exchange effect enhances the eikonal scattering phase shift χ_{E} for the elastic electron–ion collision in quantum plasmas. This expression of the total eikonal scattering phase shift χ_{E} would be quite reliable for the energy domain $\bar{E} > 1$ due to the domain of the eikonal method. Figure 2 shows the scaled differential eikonal collision cross section $\partial\bar{\sigma}_{\mathrm{E}}$ $[\equiv (\mathrm{d}\sigma_{\mathrm{E}}/\mathrm{d}\bar{b})/\pi a_Z^2]$ in units of πa_Z^2 for the elastic electron–ion collision including the electron exchange and quantum shielding effects as a function of the scaled impact parameter $\bar{b}$ for various values of the electron-exchange parameter β. As it is seen, it is found that the differential eikonal collision cross section $\partial\bar{\sigma}_{\mathrm{E}}$ increases with an increase of the electron-exchange parameter β. It is also shown in Figure 2 that the maximum position of the differential eikonal collision cross section $\partial\bar{\sigma}_{\mathrm{E}}$ is found to be receded from the collision center with increasing electron-exchange effect. We can understand that the effective inverse screening length k_+ and parameter ξ would be decreased and, however, the effective inverse screening length k_- would be increased with an increase of the electron-exchange effect so that the influence of the electron exchange weakens the electron–ion interaction in quantum plasmas. Hence, we have found that the influence of the electron exchange shifts the maximums for the differential eikonal collision cross section and also broadens the domain of the elastic electron–ion collision process in degenerate quantum plasmas. As we expect that the expression of the differential eikonal collision cross $\partial\bar{\sigma}_{\mathrm{E}}$ is also quite reliable for the energy range $\bar{E} > 1$ due to the applicability of the second-order eikonal analysis. Figure 3 represents the scaled total eikonal collision cross section $\bar{\sigma}_{\mathrm{E}}$ $(\equiv \sigma_{\mathrm{E}}/\pi a_Z^2)$ in units of πa_Z^2 for the elastic electron–ion collision in quantum plasmas as a function of the scaled collision energy $\bar{E}$ for various values of the electron-exchange parameter β. As shown in Figure 3, it is found that the electron-exchange effect enhances the total eikonal collision cross section $\bar{\sigma}_{\mathrm{E}}$. It is also found that the influence of the electron exchange on the total eikonal collision cross section $\bar{\sigma}_{\mathrm{E}}$ decreases with an increase of the collision energy $\bar{E}$. Thus, we can expect that the electron-exchange effect on the electron–ion collision process

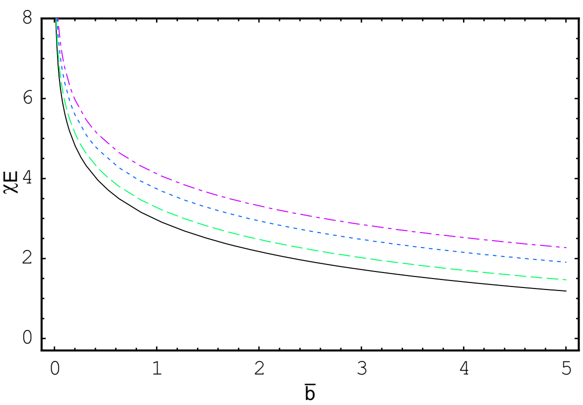

Fig. 1 (colour online). Total eikonal scattering phase shift χ_{E} for the elastic electron–ion collision as a function of the scaled impact parameter $\bar{b}$ for various values of the electron-exchange parameter β when $\Gamma\theta r_{\mathrm{s}} = 3.33$, $\bar{E} = 3$, $\bar{E}_{\mathrm{P}} = 0.08$, and $\bar{E}_{\mathrm{F}} = 0.6$. Solid line: $\beta = 0$; dashed line: $\beta = 0.5$; dotted line: $\beta = 1$; dot-dashed line: $\beta = 1.5$.

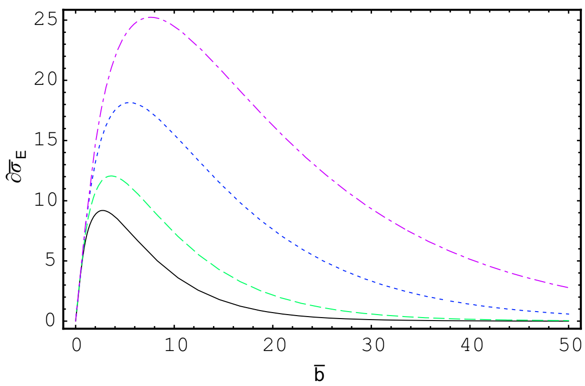

Fig. 2 (colour online). Scaled differential eikonal collision cross section $\partial\bar{\sigma}_{\mathrm{E}}$ in units of πa_Z^2 for the elastic electron–ion collision as a function of the scaled impact parameter $\bar{b}$ when $\Gamma\theta r_{\mathrm{s}} = 3.33$, $\bar{E} = 5$, $\bar{E}_{\mathrm{P}} = 0.08$, and $\bar{E}_{\mathrm{F}} = 0.6$. Solid line: $\beta = 0$; dashed line: $\beta = 0.5$; dotted line: $\beta = 1$; dot-dashed line: $\beta = 1.5$.

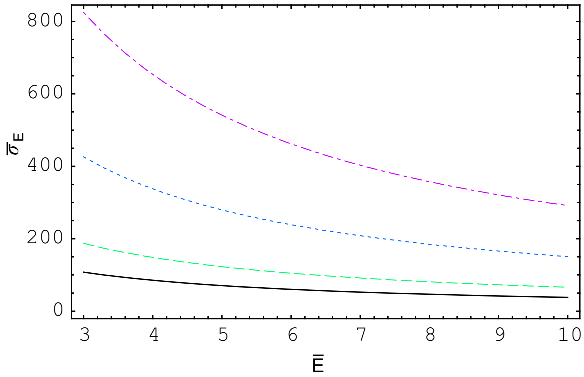

Fig. 3 (colour online). Scaled total eikonal collision cross section $\bar{\sigma}_E$ in units of πa_Z^2 for the elastic electron–ion collision as a function of the scaled collision energy $\bar{E}$ when $\Gamma\theta r_s = 3.33$, $\bar{E}_P = 0.08$ and $\bar{E}_F = 0.6$. Solid line: $\beta = 0$; dashed line: $\beta = 0.5$; dotted line: $\beta = 1$; dot-dashed line: of $\beta = 1.5$.

would be more effectively investigated in the intermediate domain of the collision energy. Figure 4 represents the surface plot of the scaled total eikonal collision cross section $\bar{\sigma}_E$ for the elastic electron–ion collision as a function of the scaled plasmon energy $\bar{E}_p$ and electron-exchange parameter β. From this figure, it is found that the eikonal collision cross section $\bar{\sigma}_E$ decreases with an increase of the plasmon energy $\bar{E}_p$. It is also found that the plasmon energy effect on the elastic electron–ion collision process decreases with increasing electron-exchange parameter β. In addition, the dependence of the electron-exchange effect in the total eikonal collision cross section $\bar{\sigma}_E$ is found to be more significant when the exchange velocity is greater than the Fermi velocity. Figure 5 shows the surface plot of the scaled total eikonal collision cross section $\bar{\sigma}_E$ for the elastic electron–ion collision as a function of the scaled Fermi energy $\bar{E}_F$ and electron-exchange parameter β. As it is seen from Figure 5, it is found that the eikonal collision cross section $\bar{\sigma}_E$ increases with increasing Fermi energy $\bar{E}_F$. In addition, it is found that the Fermi energy effect on the total eikonal collision cross section increases with an increase of the electron-exchange effect. Hence, we have also understood that the influence of the Fermi energy on the elastic electron–ion collision process in quantum plasmas would be more effectively explored in intermediate β domains. Moreover, the dependence of the electron-exchange effect in the eikonal collision cross section $\bar{\sigma}_E$ is found to be more effective when $\bar{E}_F > 0.5$. In this work, we have found that the influence of the electron exchange and quantum shielding plays an important role on the elastic electron–ion collision process in quantum plasmas. These results would provide useful information on the physical characteristics of the collision processes in quantum plasmas and also on the physical properties of degenerate quantum plasmas including the electron exchange and quantum shielding effects.

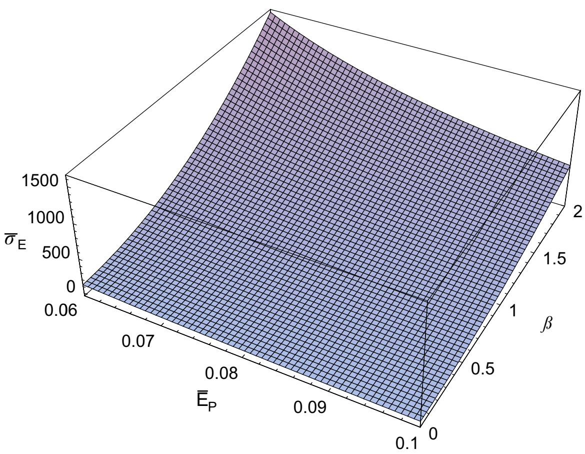

Fig. 4 (colour online). Surface plot of the scaled total eikonal collision cross section $\bar{\sigma}_E$ for the elastic electron–ion collision as a function of the scaled plasmon energy $\bar{E}_P$ and electron-exchange parameter β when $\Gamma\theta r_s = 3.33$, $\bar{E} = 5$, and $\bar{E}_F = 0.6$.

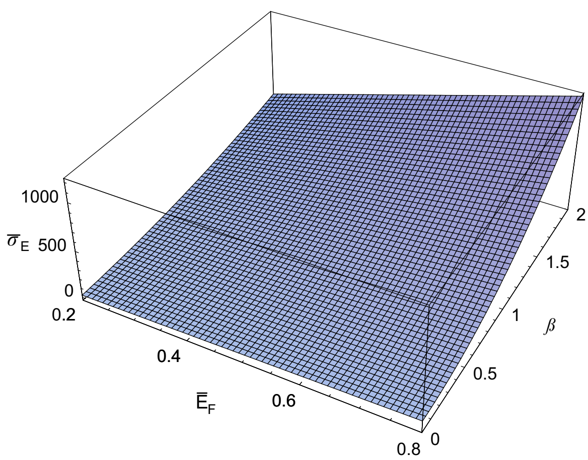

Fig. 5 (colour online). Surface plot of the scaled total eikonal collision cross section $\bar{\sigma}_E$ for the elastic electron–ion collision as a function of the scaled Fermi energy $\bar{E}_F$ and electron-exchange parameter β when $\bar{E} = 5$ and $\bar{E}_P = 0.08$.

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