# Interaction Behaviours Among Special Solitons in the (2+1)-Dimensional Modified Dispersive Water-Wave System

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Z. Naturforsch. **68a,** 447 – 453 (2013) / DOI: 10.5560/ZNA.2013-0025
Received November 20, 2012 / revised February 5, 2013 / published online May 1, 2013

A modified mapping method is used to obtain variable separation solutions with two arbitrary functions of the $(2+1)$-dimensional modified dispersive water-wave system. Based on the variable separation solution and by selecting appropriate functions, we discuss interaction behaviours among special anti-solitons constructed by multi-valued functions. The analysis results exhibit that the interaction behaviours among special anti-dromion, dromion-like anti-peakon, and dromion-like anti-semifoldon are all non-completely elastic and phase shifts exist, while the interaction behaviour among dromion-like anti-semifoldons is completely elastic and without phase shifts.

*Key words:* Modified Dispersive Water-Wave System; Modified Mapping Method; Positive and Negative Symmetric Variable Separation Solution; Interactions Between Special Solitons.

*PACS numbers:* 05.45.Yv; 02.30.Ik; 03.65.Ge

## 1. Introduction

In recent decades, there has been noticeable progress in the study of the soliton theory. Many important phenomena and dynamic processes, in almost all branches of physics like the fluid dynamics, plasma physics, field theory, nonlinear optics, and condensed matter physics, etc., are governed by the nonlinear evolution equations (NLEEs). Therefore constructing possible exact solutions [1 – 5] to a NLEE arising from the field of mathematical physics is a popular topic, but solving nonlinear physics problems is much more difficult than solving the linear ones.

In contrast with linear wave theory where one can make use of the basic technique of Fourier analysis and the variable separation approach (VSA), the celebrated inverse scattering transformation and VSA also play an important role in the nonlinear domain. As an important VSA, the multilinear variable separation approach (MLVSA) has been established and extensively applied to solve various NLEEs [6]. Along the idea of MLVSA, the mapping method, which is usually used to search for travelling wave solutions, is extended to obtain variable separation solutions of various NLEEs. The VSA based on the mapping method was firstly presented by Zheng et al. [7] and developed into $(1+1)$-dimensional and $(3+1)$-dimensional NLEEs [8]. Then many direct methods based on different mapping equations, including the extended tanh-function method (ETM) [9, 10], the improved projective approach [11], the $q$-deformed hyperbolic functions method [12], and the projective Ricatti equation method (PREM) [13], were chosen to realize the variable separation to nonlinear equations.

Many single-valued localized structures (dromions, peakons, and compactons etc.) have been extensively investigated [6 – 9, 11 – 13]. However, in the real natural phenomena, there exist very complicated folded phenomena such as the folded protein [14], folded brain and skin surfaces, and many other kinds of folded biologic systems [15]. Moreover, semifolded structures can also be realized. For example, ocean waves may fold in one direction, say $x$, and localize in a usual single valued way in another direction, say $y$. These special localized structures can be constructed by multi-valued functions. Of course, at the present stage, it is impossible to make satisfactory analytic descriptions for such complicated folded natural phenomena. However, it is still worth starting with some simpler cases. For example, some combined structures of

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dromions, peakons, and foldons and interaction among them hardly are reported.

Now some significant and interesting issues arise: Are there other mapping equations that can be used to obtain variable separation solutions of some $(2+1)$-dimensional nonlinear physical systems? Based on these variable separation solutions, can we discuss some new dynamical behaviours among some combined structures of dromions, peakons, and foldons? Motivated by these issues, we will report and discuss these phenomena in the following well-known $(2+1)$-dimensional modified dispersive water-wave (MDWW) system

$$\begin{aligned} &u_{ty}+u_{xxy}-2v_{xx}-(u^2)_{xy}=0\,,\\ &v_t-v_{xx}-2(uv)_x=0\,, \end{aligned} \tag{1}$$

which was used to model nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth. It may be derived from the inner parameter-dependent symmetry constraint of the celebrated Kadomtsev–Petviashvili (KP) equation [16]. It is worth mentioning that this system has been widely applied in many branches of physics like plasma physics, fluid dynamics, nonlinear optics, etc. Therefore, a good understanding of more solutions of the MDWW system (1) is very helpful, especially for coastal and civil engineers to apply the nonlinear water model in a harbour and coastal design. Abundant propagating localized excitations were derived by Tang et al. [6] with help of a Painlevé–Bäcklund transformation and a MLVSA. Folded localized excitations were also revealed in [17]. Some soliton fusion and fission phenomena of the MDWW system (1) have been discussed [13].

The paper is organized as follows. In Section 2, the modified mapping method is presented. The variable separation solution of a $(2+1)$-dimensional MDWW is obtained in Section 3. In Section 4, completely and non-completely elastic interaction phenomena among special solions are investigated. A brief discussion and summary is given in the last section.

## 2. The Modified Mapping Method

For a given NLEE with independent variables $x=(x_0=t,x_1,x_2,x_3,\ldots,x_m)$ and dependent variable $u$,

$$L(u,u_t,u_{x_i},u_{x_ix_j},\ldots)=0\,, \tag{2}$$

where $L$ is in general a polynomial function of its argument, and the subscripts denote the partial derivatives.

The basic idea of the mapping method is to seek for its ansätz with positive and negative symmetric form

$$u=a_0(x)+\sum_{i=1}^{n}\left\{a_i(x)\phi^i[q(x)]+a_{-i}(x)\phi^{-i}[q(x)]\right\}, \tag{3}$$

where $a_0$, $a_i$, and $a_{-i}$ are arbitrary functions of $\{x\}$ to be determined, and $n$ is fixed by balancing the linear term of the highest order with the nonlinear term in (2), $\phi$ satisfying many mapping equations, such as the Riccati equation $\phi'=l_0+\phi^2$ ($l_0$ is a real constant and the prime denotes differentiation with respect to $q$) [9], $\phi'=\sigma\phi+\phi^2$ ($\sigma$ is a real constant) [18], and $\phi'=l_1+l_2\phi^2$ ($l_1$ and $l_2$ are two real constants) [19].

Here we seek for its solution of the given NLEE (2) with the following mapping equation [20]:

$$\phi'=(A\phi-a)(B\phi-b)\,, \tag{4}$$

which is known to possess the general solution

$$\phi=\frac{b\exp[(aB-Ab)q]-a\exp[C_1(Ab-aB)]}{B\exp[(aB-Ab)q]-A\exp[C_1(Ab-aB)]}\,. \tag{5}$$

Here $C_1$ is an integration constant, further, $A$, $B$, $a$, and $b$ are arbitrary constants.

To determined $u$ explicitly, we take following three steps:

Step 1: Determine $n$ by balancing the highest nonlinear terms and the highest-order partial differential terms in the given NLEE (2).

Step 2: Substituting (3) along with (4) into (2) yields a set of polynomials for $\phi^i$. Eliminating all the coefficients of the powers of $\phi^i$, yields a series of partial differential equations, from which the parameters $a_0$, $a_i$, $a_{-i}$, and $q$ are explicitly determined.

Step 3: Substituting $a_0$, $a_i$, $a_{-i}$, $q$, and (5) into (3), one can obtain possible solutions of (2).

**Remark 1.** It seems that the mapping equation (4) is a new equation. However, when we re-define $\phi\equiv\phi-\frac{Ab+aB}{2AB}$ and $l_0=-\frac{A^2b^2+a^2B^2}{A^2B^2}$, (4) can be transformed to the known mapping equation $\phi'=l_0+\phi^2$, which possesses the following solutions [7, 8]

$$\phi = \begin{cases} -\sqrt{-l_0}\tanh(\sqrt{-l_0}q) \ \text{for} \ l_0 < 0, \\ -\sqrt{-l_0}\coth(\sqrt{-l_0}q) \ \text{for} \ l_0 < 0, \\ \sqrt{l_0}\tan(\sqrt{l_0}q) \ \text{for} \ l_0 > 0, \\ -\sqrt{l_0}\cot(\sqrt{l_0}q) \ \text{for} \ l_0 > 0. \end{cases} \quad (6)$$

**Remark 2.** Although the mapping equation (4) can be transformed to the known mapping equation, solution (5) contains solutions (6). When we choose $C_1 = 0$, $a = b = -\sqrt{-l_0}$, and $-A = B = -1$ in solution (5), the first solution in (6) can be obtained. If one takes $C_1 = 0, A = B = 1$, and $a = -b = \sqrt{-l_0}$ in solution (5), the second solution in (6) can be derived. When one selects $C_1 = 0$, $a = b = -I\sqrt{l_0}$, and $-A = B = -1$ in solution (5), the third solution in (6) can be recovered. Moreover, if we set $C_1 = 0$, $A = B = 1$, and $a = -b = \mathrm{i}\sqrt{l_0}$ in solution (5), the last solution in (6) can be obtained. Therefore, solution (5) is more general than solutions (6).

## 3. Variable Separation Solutions for the (2+1)-Dimensional MDWW Equation

To solve the $(2+1)$-dimensional MDWW system, first, let us make a transformation for (1): $v = u_y$. Substituting the transformation into (1) yields

$$u_{ty} - 2(u_x u)_y - u_{xxy} = 0. \quad (7)$$

Along with the modified mapping method in Section 2, by balancing the higher-order derivative terms with the nonlinear terms in (7), we suppose that (7) has the following formal solutions:

$$u(x,y,t) = a_0(x,y,t) + a_1(x,y,t)\phi(q) + \frac{a_{-1}(x,y,t)}{\phi(q)}, \quad (8)$$

where $\phi$ satisfies (5) and $q \equiv q(x,y,t)$. Here we select the variable separation ansätz [13]

$$q = \chi(x,t) + \psi(y), \quad (9)$$

which implies that two spatial variable $x$ and $y$ are separated completely. Inserting (8) with (9) into (7), and eliminating all the coefficients of the powers of $\phi^i$, one gets a set of partial differential equations. It is very difficult to solve these prolix and complicated differential equations. Fortunately, by careful analysis and calculation, we derive two special solutions, namely Solution 1

$$a_0 = \frac{(Ab+aB)\chi_x^2 - \chi_{xx} + \chi_t}{2\chi_x}, \quad a_1 = -AB\chi_x, \ a_{-1} = 0, \quad (10)$$

and Solution 2

$$a_0 = \frac{\chi_t - \chi_{xx}}{2\chi_x}, \ a_1 = -AB\chi_x, \quad a_{-1} = ab\chi_x, \ Ab + aB = 0, \quad (11)$$

where $\chi$ and $\psi$ are arbitrary functions of $\{x,t\}$ and $\{y\}$, respectively.

Therefore, the variable separation solutions of the $(2+1)$-dimensional WDMM system read

$$u = \frac{(Ab+aB)\chi_x^2 - \chi_{xx} + \chi_t}{2\chi_x} - AB\chi_x \cdot \frac{b\exp\left[(aB-Ab)(\chi+\psi)\right] - a\exp\left[C_1(Ab-aB)\right]}{B\exp\left[(aB-Ab)(\chi+\psi)\right] - A\exp\left[C_1(Ab-aB)\right]}, \quad (12)$$

$$v = -abAB\chi_x\psi_y + (Ab+aB)AB\chi_x\psi_y \cdot \frac{b\exp\left[(aB-Ab)(\chi+\psi)\right] - a\exp\left[C_1(Ab-aB)\right]}{B\exp\left[(aB-Ab)(\chi+\psi)\right] - A\exp\left[C_1(Ab-aB)\right]} - A^2B^2\chi_x\psi_y \left\{\frac{b\exp\left[(aB-Ab)(\chi+\psi)\right] - a\exp\left[C_1(Ab-aB)\right]}{B\exp\left[(aB-Ab)(\chi+\psi)\right] - A\exp\left[C_1(Ab-aB)\right]}\right\}^2, \quad (13)$$

and

$$u = \frac{\chi_t - \chi_{xx}}{2\chi_x} - Ab\chi_x \cdot \frac{A\exp\left[2AbC_1\right] + B\exp\left[-2Ab(\chi+\psi)\right]}{A\exp\left[2AbC_1\right] - B\exp\left[-2Ab(\chi+\psi)\right]} + ab\chi_x \cdot \frac{b\left\{A\exp\left[2AbC_1\right] - B\exp\left[-2Ab(\chi+\psi)\right]\right\}}{B\left\{A\exp\left[2AbC_1\right] + B\exp\left[-2Ab(\chi+\psi)\right]\right\}}, \quad (14)$$

$$v = 2A^2b^2B\chi_x\psi_y\Big(\exp\left[-2Ab(\chi+\psi)\right]\Big) \cdot \Big(A\exp\left[2AbC_1\right] - B\exp\left[-2Ab(\chi+\psi)\right]\Big)^{-1} - 2A^2b^2B\chi_x\psi_y\Big(\exp\left[-2Ab(\chi+\psi)\right]\Big\{A\exp\left[2AbC_1\right] + B\exp\left[-2Ab(\chi+\psi)\right]\Big\}\Big)\Big(\Big\{A\exp\left[2AbC_1\right] - B\exp\left[-2Ab(\chi+\psi)\right]\Big\}\Big)^{-2} - 2abAB^2\chi_x\psi_y \quad (15)$$

$$\cdot \frac{\exp\left[-2Ab(\chi+\psi)\right]}{A\exp\left[2AbC_1\right]+B\exp\left[-2Ab(\chi+\psi)\right]}$$
$$-2abAB^2\chi_x\psi_y\Big(\exp\left[-2Ab(\chi+\psi)\right]\Big\{A\exp\left[2AbC_1\right]$$
$$-B\exp\left[-2Ab(\chi+\psi)\right]\Big\}\Big)\Big(\Big\{A\exp\left[2AbC_1\right]$$
$$+B\exp\left[-2Ab(\chi+\psi)\right]\Big\}\Big)^{-2},$$

where $\chi(x,t)$ and $\psi(y)$ are two arbitrary variable separation functions.

## 4. Interaction Behaviours Among Special Solitons

$(2+1)$-dimensional MDWW system models nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth. Single-valued line solitons used to analyze nonlinear and dispersive long gravity waves travelling in two horizontal directions. For example, we can use them to describe roughly the bubbles on (or under) a fluid surface. However, these waves are folded or semi-folded waves, and it is too complicated to use only single-valued functions to analyze the dynamical behaviours of water waves. More precisely, we can use multi-valued functions to describe them because multi-valued functions can construct folded or semi-folded structures (foldons or semi-foldons) [9].

In this section, we will pay attention to interaction behaviours between semi-foldons for the physical quantity $v$ expressed by (15). Here we use symbolic computation software MAPLE to study these behaviours. Firstly, we discuss the three special combined soliton structures, i. e. special anti-dromion, dromion-like anti-peakon, dromion-like anti-semifoldon by introducing a multi-valued function as

$$\chi_x=\sum_{i=1}^{N}\kappa_i(\zeta-d_it),\;\; x=\zeta+\sum_{i=1}^{N}\eta_i(\zeta-d_it), \tag{16}$$

where $d_i$ $(i=1,2,\ldots,N)$ are arbitrary constants, $\kappa_i$ and $\eta_i$ are localized excitations with the properties $\kappa_i(\pm\infty)=0$, $\eta_i(\pm\infty)=\text{consts}$. From (16), one can know that $\zeta$ may be a multi-valued function in some suitable regions of $x$ by choosing the functions $\eta_i$ appropriately. Therefore, the function $\chi_x$, which is obviously an interaction solution of $N$ localized excitations due to the property $\zeta|_{x\to\infty}\to\infty$, may be a multi-valued function of $x$ in these areas, though it is a single-valued function of $\zeta$. Actually, most of the known multi-loop solutions are special cases of (16).

Specifically, $\chi$ and $\psi$ are chosen as

$$\psi=1+\tanh(y), \tag{17}$$

$$\chi_x=0.5\,\mathrm{sech}^2(\zeta-0.5t),\quad x=\zeta-C\tanh(\zeta-0.5t), \tag{18}$$

where $C$ is a characteristic parameter, which determines the localized structure. Figure 1 describes these special localized structures, i. e. special anti-dromion, dromion-like anti-peakon, dromion-like anti-semifoldon with $C=0.5$, 0.95, 1.5, respectively. They localize as anti-bell-like soliton in the $y$-direction and anti-bell-like soliton, anti-peakon, and anti-loop soliton in the $x$-direction, respectively.

Next, let us study interaction behaviours among these special anti-solitons produced by the multi-valued functions above. If we take the specific choice

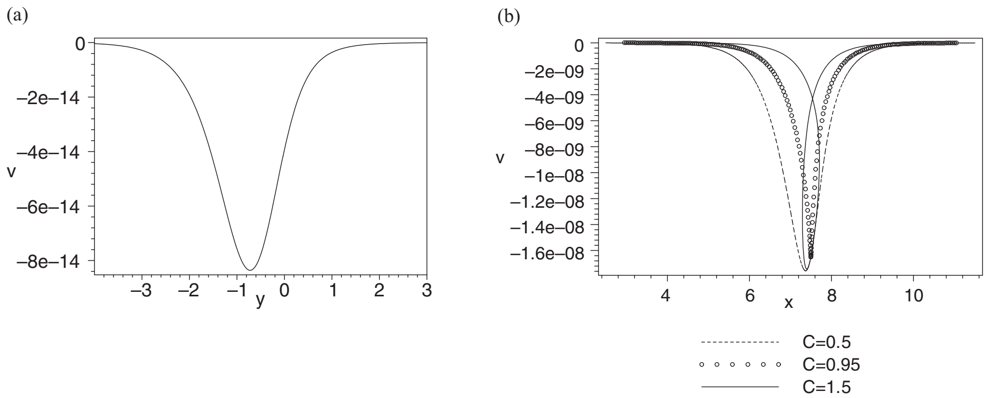


Fig. 1. Sectional views of special solitons at (a) $x=0$ and (b) $y=0$ for parameters $A=-2$, $B=C_1=1$, $a=0.5$, $b=0.25$ at time $t=15$.

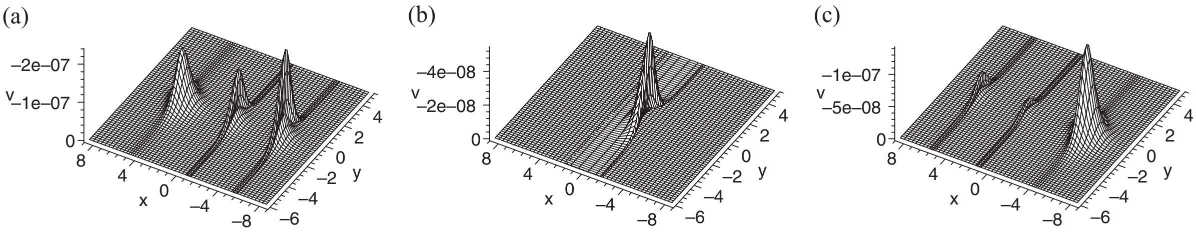


Fig. 2. Incompletely elastic interaction among two dromion-like anti-semifoldons and one special anti-dromion at time (a) $t=-15$, (b) $t=-0.1$, and (c) $t=15$. The parameters are chosen as $A=-2$, $B=C_1=1$, $a=0.5$, $b=0.25$, $C=D=1.5$, $E=0.5$.

$N=3$, $d_1=0$, $d_2=0.5$, and $d_3=-0.5$ in (16), one has

$$\begin{aligned}\chi_x &= 0.3\,\text{sech}^2(\zeta)+0.5\,\text{sech}^2(\zeta-0.5t)\\ &\quad +0.7\,\text{sech}^2(\zeta+0.5t)\,,\\ x &= \zeta - C\tanh(\zeta)-D\tanh(\zeta-0.5t)\\ &\quad -E\tanh(\zeta+0.5t)\,,\end{aligned} \tag{19}$$

where $C$, $D$, and $E$ are characteristic parameters, which determine the types of interaction. Moreover, $\psi$ is given by (17). From the expression $v$ with (19) and (15), one can obtain three solitons, one is static, another is moving along positive $x$-direction, and the last one is moving along negative $x$-direction. Note that it is for the first time that three special anti-solitons produced by multi-valued functions are studied analytically and graphically.

The interactions between solitons may be regarded as elastic or inelastic. It is called completely elastic, if the amplitude, velocity, and wave shape of the solitons do not changed after their interaction. Otherwise, the interaction between solitons is inelastic (non-completely elastic and completely non-elastic). Like the collisions between two classical particles, a collision in which the solitons stick together is sometimes called completely inelastic.

Firstly, if we take the specific values $C=D=1.5$, $E=0.5$ in (19), then we can successfully construct an interaction among two dromion-like anti-semifoldons and one special anti-dromion, of which possess a phase shift for the physical quantity $v$ depicted in Figure 2. From Figure 2, one can find that the interaction may exhibit a incompletely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction. Moreover, we can see that this interaction possesses a novel property, namely, there exists a multi-valued semi-foldon in the process of their collision, which is different from the reported case among three single-valued structures in previous literature [21].

The phase shift can also be observed. Prior to interaction, the velocities of the smaller anti-semifoldon, the special anti-dromion, and the larger anti-semifoldon have set to be $\{v_{01x}=d_1=0\}$, $\{v_{02x}=d_2=0.5\}$, and $\{v_{03x}=d_3=-0.5\}$, respectively. The smaller anti-semifoldon site changes from $x=-1$ to $x=1$, then resides at $x=1$ and maintains its initial velocity $\{v_{1x}=v_{01x}=0\}$ (i. e. static) after interaction. Therefore the magnitude of the phase shift of the static smallest anti-semifoldon is 3. The final velocities $v_{2x}$ and $v_{3x}$ of the moving larger solitons also completely maintain their initial velocities $\{v_{2x}=v_{02x}=0.5\}$ and $\{v_{3x}=v_{03x}=-0.5\}$.

Similarly, we can discuss the interaction among one dromion-like anti-semifoldons and two special anti-dromion by setting the specific values $C=0.5$, $D=1.5$, and $E=0.5$ in (19). This case is still a non-completely elastic interaction. For the limit of length, we omit the detailed discussion about it.

Secondly, if we choose the specific values $C=0.95$, $D=1.5$, and $E=0.95$ in (19), then we can successfully obtain an interaction among two dromion-like anti-peakons and one dromion-like anti-semifoldon. This interaction is also a non-completely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction (see Fig. 3). Different from interactions among three single-valued structures [21], the multi-valued semi-foldon also appears in the process of their collision. Through careful analysis similar to that in Figure 2, we know that the phase shift of the static smaller dromion-like anti-peakon is 1.1. The smaller dromion-like anti-peakon and the moving larger dromion-like anti-peakon and dromion-like anti-semifoldon maintain their initial velocities $\{v_{1x}=v_{01x}=0\}$ (i. e. static), $\{v_{2x}=v_{02x}=0.5\}$, and $\{v_{3x}=$

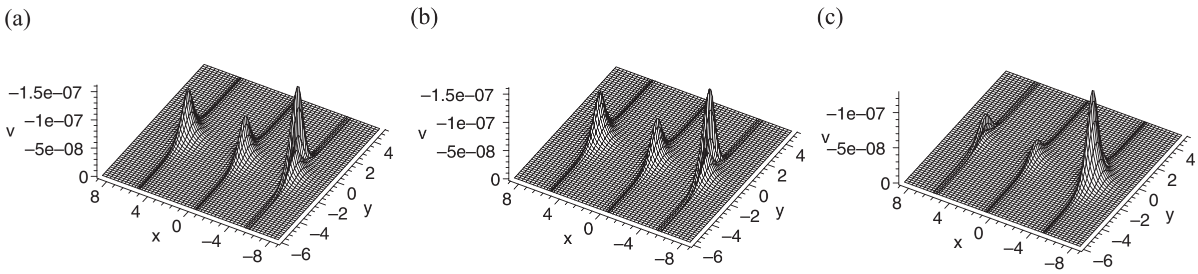


Fig. 3. Non-completely elastic interaction among two dromion-like anti-peakons and one dromion-like anti-semifoldon at time (a) $t=-15$, (b) $t=-1$, and (c) $t=15$. The parameters are chosen as $A=-2$, $B=C_1=1$, $a=0.5$, $b=0.25$, $C=0.95$, $D=1.5$, $E=0.95$.

$v_{03x}=-0.5\}$, respectively. Of course, we can investigate non-completely elastic behaviour among one dromion-like anti-peakons and two dromion-like anti-semifoldon by selecting $C=0.95$, $D=1.5$, and $E=1.5$. Here we still omit it for the limit of length.

In the following, when we set the specific values $C=1.5$, $D=0.5$, and $E=0.95$ in (19), we can discuss the interaction among special anti-dromion, dromion-like anti-peakon, and dromion-like anti-semifoldon. This interaction is also a non-completely elastic behaviour since solitons' shapes and amplitudes are not completely maintained any more after interaction (see Fig. 4). Similarly to two cases above, the semi-foldon exists again during the interaction among these solitons. Through careful analysis similar to that in Figure 2, we know that the phase shift of the static dromion-like anti-semifoldon is 0.88. The dromion-like anti-semifoldon and the moving special anti-dromion and dromion-like anti-peakon also preserve their initial velocities.

Finally, it is interesting to note that although the above selections are all non-completely elastic interaction behaviours, we can also construct localized coherent structures with completely elastic interaction behaviours by appropriately selecting the values of $C$, $D$, and $E$ in (19).

If we select the specific values $C=D=E=1.5$ in (19), then we can successfully construct the interaction among three dromion-like anti-semifoldons for the physical quantity $v$ depicted in Figure 5. From Figure 5, one can find that the interaction among them may exhibit a completely elastic behaviour since solitons' shapes, amplitudes, and velocities are completely maintained after interaction. The phase shift is not observed. Before interaction, the static smallest semifoldon is located at $x=0$ and after the interaction, it is still located at $x=0$ and then resides at $x=0$ and maintains its initial velocities $\{v_{1x}=v_{01x}=0\}$. While the moving larger semifoldons also completely maintain their initial velocities $\{v_{2x}=v_{02x}=0.5\}$ and $\{v_{3x}=v_{03x}=-0.5\}$, respectively. These properties of interaction among three dromion-like anti-semifoldons are similar to that of an interaction among three semifoldons in [22].

## 5. Summary and Discussion

In summary, a modified mapping method is presented with positive and negative symmetric ansätz form. Using this method, we obtain variable separation solutions of the $(2+1)$-dimensional MDWW system. Based on the variable separation symmetric solu-

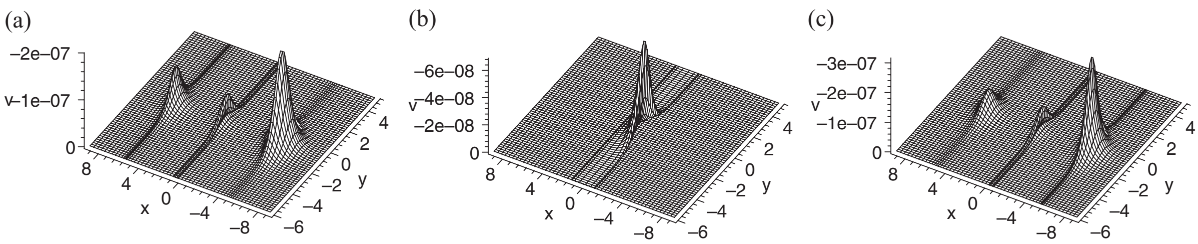


Fig. 4. Non-completely elastic interaction among special anti-dromion, dromion-like anti-peakon and dromion-like anti-semifoldon at time (a) $t=-15$, (b) $t=-1$, and (c) $t=15$. The parameters are chosen as $A=-2$, $B=C_1=1$, $a=0.5$, $b=0.25$, $C=1.5$, $D=0.5$, $E=0.95$.

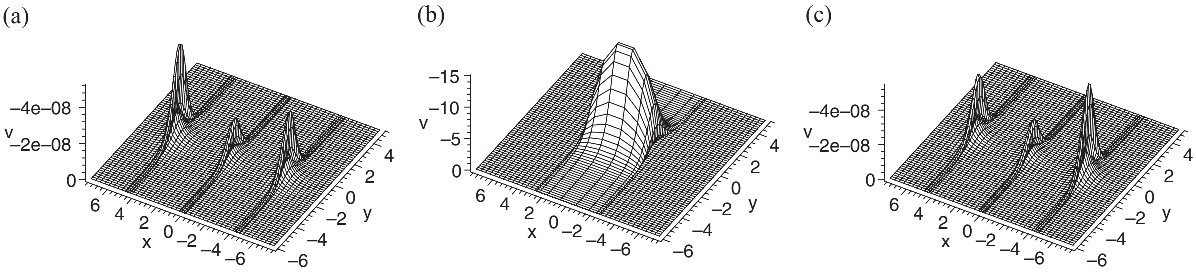

Fig. 5. Completely elastic interaction among three dromion-like anti-semifoldons at time (a) $t = -15$, (b) $t = 0$, and (c) $t = 15$. The parameters are chosen as $A = -2$, $B = C_1 = 1$, $a = 0.5$, $b = 0.25$, $C = D = E = 1.5$.

tion (15) and by selecting appropriate functions, four types of interaction behaviours between special anti-solitons, constructed by multi-valued functions, are investigated. The interaction behaviours among special anti-dromion, dromion-like anti-peakon, and dromion-like anti-semifoldon are all non-completely elastic and phase shifts exist, while the interaction behaviour among dromion-like anti-semifoldons is completely elastic and without phase shifts. Of course, there are some pending issues to be further studied. How to quantify the notion of complete or non-complete elasticity more suitably? What is the general equation for the distribution of the energy and momentum for these interactions?

We have also verified that the modified mapping method is quite concise and useful to generate abundant localized excitations. Actually, this method presented in this paper is only an initial work, more work about the method should be concerned. In our future work, we can also extend this method to other $(2+1)$-dimensional NLEEs, such as Korteweg–de Vries equation, Nizhnik–Novikov–Veselov system, dispersive long wave equation etc.

*Acknowledgements*

This work was supported by the National Natural Science Foundation of China (Grant Nos. 11005092), the Zhejiang Province Undergraduate Scientific and Technological Innovation Project (Grant No. 2012R412018) and Undergraduate Innovative Base of Zhejiang Agriculture and Forestry University.

[1] J. X. Fei and C. L. Zheng, Z. Naturforsch. **66a**, 1 (2011).
[2] S. Zhang, Y. N. Sun, J. M. Ba, and L. Dong, Z. Naturforsch. **66a**, 33 (2011).
[3] X. J. Lai, and X. O. Cai, Z. Naturforsch **66a**, 392 (2011).
[4] C. E. Ye and W. G. Zhang, Chaos Solit. Fract. **44**, 1063 (2011).
[5] D. S. Wang and Y. Liu, Z. Naturforsch **65a**, 71 (2010).
[6] X. Y. Tang, S. Y. Lou, and Y. Zhang, Phys. Rev. E **66**, 46601 (2002).
[7] C. L. Zheng, J. P. Fang, and L. Q. Chen, Z. Naturforsch **59a**, 912 (2004).
[8] C. Q. Dai, C. J. Yan, and J. F. Zhang, Commun. Theor. Phys. **46**, 389 (2006).
[9] C. Q. Dai and G. Q. Zhou, Chin. Phys. **16**, 1201 (2007).
[10] C. Q. Dai, G. Q. Zhou, and J. F. Zhang, Chaos Solit. Fract. **33**, 1458 (2007).
[11] Z. Yang, S. H. Ma, and J. P. Fang, Chin. Phys. B **20**, 40301 (2011).
[12] C. Q. Dai, Chaos Solit. Fract. **38**, 474 (2008).
[13] C. Q. Dai and Y. Y. Wang, Nonlin. Dyn. **70**, 189 (2012).
[14] S. C. Trewick, T. F. Henshaw, R. P. Hausinger, T. Lindahl, and B. Sedgwick, Nature **419**, 174 (2002).
[15] B. L. MacInnis and R. B. Campenot, Science **295**, 1536 (2002).
[16] S. Y. Lou and X. B. Hu, J. Math. Phys. **38**, 6401 (1997).
[17] Y. Lei, S. H. Ma, and J. P. Fang, Chin. Phys. B **22**, 10506 (2013).
[18] S. H. Ma, J. P. Fang, Q. B. Ren, and Z. Yang, Chin. Phys. B **21**, 50511 (2012).
[19] C. Q. Dai and Y. Y. Wang, Phys. Lett. A **372**, 1810 (2008).
[20] W. L. Chen, W. T. Zhang, L. P. Zhang, and C. Q. Dai, Chin. Phys. B **21**, 110507 (2012).
[21] C. L. Zheng, Commun. Theor. Phys. **41**, 513 (2004).
[22] C. L. Zheng, Commun. Theor. Phys. **43**, 1061 (2005).