

The Correlated Environments Depress Entanglement Decoherence in the Dimer System

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In this work, the decoherence properties of two independent dimer systems coupled to two correlated Fermi-spin environments is investigated under the non-Markovian condition. We demonstrate that the correlated spin bath can effectively depress the entanglement decoherence, and a steady entanglement can be achieved when the coupling parameter q exceeds a critical value. This result shows a possible method of the entanglement preservation in decohering environments.

Key words: Decoherence; Entanglement; Quantum Noise.

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1. Introduction

In a realistic physical system, the decoherence phenomena of the entanglement arising from the inevitable interaction between the environment and systems always exist and play a coherence-destructive role [1, 2]. Hence, how to depress the decoherence and how to instrumentally keep the system entangled under environmental noise is of paramount relevance for a number of applications in modern physics, especially the quantum information and quantum computation. Some works related to the non-equilibrium process have been carried out on this aspect in the framework of a non-Markovian dynamics, including sudden death and sudden birth of entanglement [3–8], non-Markovianity-assisted steady state entanglement [9], measure for non-Markovian behaviour of quantum processes [10], non-Markovian entanglement dynamics in the presence of system–bath coherence [11], and so on [12–22]. But these works were mainly restricted to the case of the independent environments. On the other hand, entanglement preservation for the case of correlated environments is still an open problem.

In the present work, we demonstrate that the decoherence behaviour of a dimer system can be effectively depressed by increasing the coupling parameter q between the environments and that a steady entangle-

ment can be obtained when the parameter q exceeds a critical value. This result is of particular interest in understanding the mechanisms that depress the entanglement decoherence and assist entanglement preservation in condensed matter and biomolecular systems, where non-Markovian dephasing is a dominant noise source [14, 23–28].

2. Formulation of the Problem

We consider a two-qubit system with no mutual interaction. Every qubit is made of the simplest electronic energy transfer system which can be formed by a dimer system (pseudo-spin- $\frac{1}{2}$ particles). The Hamiltonian of two independent dimer systems is given by [9, 29, 30]

$$H_d = H_{d_1} + H_{d_2} \quad (1)$$

where $H_{d_1} = \epsilon_1|1\rangle\langle 1| + \epsilon_2|2\rangle\langle 2| + J_1(|1\rangle\langle 2| + |2\rangle\langle 1|)$ and $H_{d_2} = \epsilon_3|3\rangle\langle 3| + \epsilon_4|4\rangle\langle 4| + J_2(|3\rangle\langle 4| + |4\rangle\langle 3|)$ denote the Hamiltonian of two independent dimer systems, respectively. ϵ_i and $|i\rangle$ ($i = 1, 2, 3, 4$) are the energy levels and the energy states of the dimer system. J_1 and J_2 are the amplitude of transition.

After some calculations, the eigenvalues and eigenvectors of H_d can be obtained in the absence of the environment:

$$\begin{aligned}
 E_1 &= \Delta_1 + \Delta_3 + \sqrt{\Delta_2^2 + J_1^2} + \sqrt{\Delta_4^2 + J_2^2}, \quad \Psi_1 = \left[\frac{\Delta_2 + \sqrt{\Delta_2^2 + J_1^2}}{J_1} |1\rangle + |2\rangle \right] \left[\frac{\Delta_4 + \sqrt{\Delta_4^2 + J_2^2}}{J_2} |3\rangle + |4\rangle \right], \\
 E_2 &= \Delta_1 + \Delta_3 + \sqrt{\Delta_2^2 + J_1^2} - \sqrt{\Delta_4^2 + J_2^2}, \quad \Psi_2 = \left[\frac{\Delta_2 + \sqrt{\Delta_2^2 + J_1^2}}{J_1} |1\rangle + |2\rangle \right] \left[\frac{\Delta_4 - \sqrt{\Delta_4^2 + J_2^2}}{J_2} |3\rangle + |4\rangle \right], \\
 E_3 &= \Delta_1 + \Delta_3 - \sqrt{\Delta_2^2 + J_1^2} + \sqrt{\Delta_4^2 + J_2^2}, \quad \Psi_3 = \left[\frac{\Delta_2 - \sqrt{\Delta_2^2 + J_1^2}}{J_1} |1\rangle + |2\rangle \right] \left[\frac{\Delta_4 + \sqrt{\Delta_4^2 + J_2^2}}{J_2} |3\rangle + |4\rangle \right], \\
 E_4 &= \Delta_1 + \Delta_3 - \sqrt{\Delta_2^2 + J_1^2} - \sqrt{\Delta_4^2 + J_2^2}, \quad \Psi_4 = \left[\frac{\Delta_2 - \sqrt{\Delta_2^2 + J_1^2}}{J_1} |1\rangle + |2\rangle \right] \left[\frac{\Delta_4 - \sqrt{\Delta_4^2 + J_2^2}}{J_2} |3\rangle + |4\rangle \right],
 \end{aligned} \tag{2}$$

where $\Delta_1 = \frac{\varepsilon_1 + \varepsilon_2}{2}$, $\Delta_2 = \frac{\varepsilon_1 - \varepsilon_2}{2}$, $\Delta_3 = \frac{\varepsilon_3 + \varepsilon_4}{2}$, and $\Delta_4 = \frac{\varepsilon_3 - \varepsilon_4}{2}$.

The aim of this paper is to show, in general conditions $\varepsilon_1 \neq \varepsilon_2$ and $\varepsilon_3 \neq \varepsilon_4$, the dynamic evolution of the entanglement for the two dimers in contact with the same two spin environments. The Hamiltonian of the total system has the following form [5, 9, 29, 30]:

$$\begin{aligned}
 H &= H_d + \sum_{i=1,2} H_{B_i} + H_{d_1 B_1} + H_{d_1 B_2} + H_{d_2 B_1} + H_{d_2 B_2} \\
 &\quad + q \sum_{k=1}^{N_1} \sum_{k'=1}^{N_2} \frac{\sigma_z^{k,1}}{2} \frac{\sigma_z^{k',2}}{2}.
 \end{aligned} \tag{3}$$

Here, each environment B_i consists of N_i particles ($i = 1, 2$) with spin $\frac{1}{2}$:

$$H_{B_i} = \alpha_i \sum_{k=1}^{N_i} \frac{\sigma_z^{k,i}}{2}, \tag{4}$$

where $\sigma_z^{k,i}$ are the Pauli matrices, and α_i is the frequency of $\sigma_z^{k,i}$.

Because a dimer system is a system made of two subsystems (for example, two spin-like particles or two qubits) by the typically bound to each other, we consider two subsystems (every subsystem is a pseudo-spin- $\frac{1}{2}$ particles) in different environments [31], respectively. The interaction between the subsystem and the environment can be described by $\gamma \frac{\sigma_z}{2} \sum_{k=1}^{N_i} \frac{\sigma_z^{k,i}}{2}$ [5] (where γ is the coupling constant, and σ_z is the pseudo-spin Pauli operator which describes the pseudo-spin- $\frac{1}{2}$ particles). Here we only consider the interaction between the upper level and the environment [9, 29–31],

so the interaction between the dimer and the environment is described by [9, 29–31]

$$\begin{aligned}
 H_{d_1 B_1} &= \sum_{k=1}^{N_1} \gamma_1 |1\rangle \langle 1| \frac{\sigma_z^{k,1}}{2}, \quad H_{d_1 B_2} = \sum_{k=1}^{N_2} \gamma_2 |2\rangle \langle 2| \frac{\sigma_z^{k,2}}{2}, \\
 H_{d_2 B_1} &= \sum_{k=1}^{N_1} \gamma_3 |3\rangle \langle 3| \frac{\sigma_z^{k,1}}{2}, \quad H_{d_2 B_2} = \sum_{k=1}^{N_2} \gamma_4 |4\rangle \langle 4| \frac{\sigma_z^{k,2}}{2}.
 \end{aligned} \tag{5}$$

The last term $q \sum_{k=1}^{N_1} \sum_{k'=1}^{N_2} \frac{\sigma_z^{k,1}}{2} \frac{\sigma_z^{k',2}}{2}$ of (3) describes an Ising-type correlation between the environments with strength q . The cases $q = 0$ and $q \neq 0$ describe independent and correlated spin bath, respectively.

In order to obtain the exact solution of the system, we define collective spin operators $S_i^z = \sum_{k=1}^{N_i} \frac{\sigma_z^{k,i}}{2}$. Thus, the total Hamiltonian can be written as

$$\begin{aligned}
 H &= H_{d_1} + H_{d_2} + \sum_{i=1,2} \alpha_i S_i^z + q S_1^z S_2^z, \\
 H_{d_1} &= (\varepsilon_1 + \gamma_1 S_1^z) |1\rangle \langle 1| + (\varepsilon_2 + \gamma_2 S_2^z) |2\rangle \langle 2| \\
 &\quad + J_1 (|1\rangle \langle 2| + |2\rangle \langle 1|), \\
 H_{d_2} &= (\varepsilon_3 + \gamma_3 S_1^z) |3\rangle \langle 3| + (\varepsilon_4 + \gamma_4 S_2^z) |4\rangle \langle 4| \\
 &\quad + J_2 (|3\rangle \langle 4| + |4\rangle \langle 3|).
 \end{aligned} \tag{6}$$

We introduce an orthonormal basis in the bath Hilbert space H_B consisting of states $|j, m\rangle$ [5]. These states are defined as eigenstates of S_z and of S^2 and

$$\begin{aligned}
 S^2 |j, m\rangle &= j(j+1) |j, m\rangle, \\
 S_z |j, m\rangle &= m |j, m\rangle, \quad S^2 = S_x^2 + S_y^2 + S_z^2
 \end{aligned} \tag{7}$$

with $j = 0, \dots, \frac{N}{2}$, $m = j, \dots, -j$.

The formal solution of the von Neumann equation

$$\frac{d}{dt}\rho(t) = \mathcal{L}\rho(t) = -i[H, \rho(t)] \quad (8)$$

can then be written as

$$\rho(t) = e^{\mathcal{L}t}\rho(0), \quad (9)$$

where $\rho(t)$ denotes the density matrix of the total system.

Our main goal is to derive the dynamics of the reduced density matrix $\rho_d(t)$.

$$\rho_d(t) = \text{Tr}_B(e^{\mathcal{L}t}\rho(0)) \quad (10)$$

where Tr_B denotes the partial trace taken over the Hilbert space of the spin bath.

For the initial state $\rho(0) = \rho_d(0) \otimes \rho_B(0)$, the density matrix $\rho_d(t)$ of the dimer system is

$$\begin{aligned} \rho_d(t) &= \text{Tr}_B[U(t)\rho_d(0)\rho_B(0)U^\dagger(t)] \quad (11) \\ &= \frac{1}{Z} \sum_{j_1=0}^{N_1/2} \sum_{m_1=-j_1}^{j_1} \sum_{j_2=0}^{N_2/2} \sum_{m_2=-j_2}^{j_2} \frac{v(N_1, J_1)v(N_2, J_2)}{e^{\beta\alpha_1 m_1} e^{\beta\alpha_2 m_2} e^{\beta q m_1 m_2}} \\ &\times A^\dagger D A \end{aligned}$$

Here, the bath is given as the canonical distribution

$$\rho_B(0) = \frac{1}{Z} e^{g\beta s_1^z s_2^z} \prod_{i=1}^2 e^{-\beta\alpha_i s_i^z} \quad (12)$$

with $\beta = \frac{1}{K_B T}$ (K_B is Boltzmann constant, T is temperature) and the initial condition $\rho_d(0) = A^\dagger G A$, where A , D , and G denote the matrices which are given in the Appendix.

The partition function Z of the bath is

$$Z = \sum_{j_1=0}^{N_1/2} \sum_{m_1=-j_1}^{j_1} \sum_{j_2=0}^{N_2/2} \sum_{m_2=-j_2}^{j_2} \frac{v(N_1, J_1)v(N_2, J_2)}{e^{\beta\alpha_1 m_1} e^{\beta\alpha_2 m_2} e^{\beta q m_1 m_2}}, \quad (13)$$

where $v(N_i, J_i)$ denotes the degeneracy of the spin bath [5, 32, 33].

To quantify the entanglement, we use the Wootters concurrence [34], defined as $C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}$, where λ_l ($l = 1, 2, 3, 4$) are the square roots of the eigenvalues of the matrix $R = \rho^{1/2}(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)\rho^{1/2}$ in decrease order, and ρ is the density matrix expressed in the standard basis $|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$. σ_y is the normal Pauli matrix, and the asterisk indicates complex conjugation.

3. Results

After some calculation, we can obtain the evolution of the entanglement states, as shown in Figure 1 and Figure 2.

In Figure 1, we show the entanglement dynamics of two qubits for the initial state A (disentanglement state) with parameter q and time t . From Figure 1, for the smaller parameter q , the entanglement does not exhibit sudden birth. When we adjust the coupling strength between the spin baths, namely the larger parameter q , the entanglement appears. The time evolution of entanglement presents the phenomenon of sudden birth and sudden death and shows the non-periodic oscillation evolution.

In Figure 2, it is shown that the entanglement dynamics of two qubits for the entanglement initial state changes with parameter q and time t . The time evolution of entanglement presents the phenomenon of sudden birth and sudden death even if the initial state is the entanglement state, and shows the oscillation evolution. The most important and interesting nature is effectively depressing the entanglement decoherence by increasing the parameter q . Especially, for the smaller concurrence [34] $C(\rho)$ of the initial state, we must use larger q to depress the decoherence. The time evo-

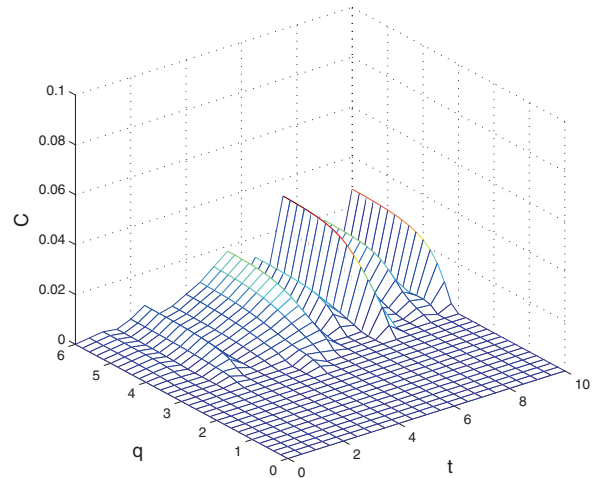


Fig. 1 (colour online). Change of the concurrence for the initial state $\phi_d(0) = \frac{\sqrt{2}}{4}|1\rangle|3\rangle + \frac{\sqrt{2}}{4}|2\rangle|3\rangle + \frac{\sqrt{6}}{4}|1\rangle|4\rangle + \frac{\sqrt{6}}{4}|2\rangle|4\rangle$. The parameters are: $\gamma_1 = 0.2 \text{ ps}^{-1}$, $\gamma_2 = 0.3 \text{ ps}^{-1}$, $\gamma_3 = 0.4 \text{ ps}^{-1}$, $\gamma_4 = 0.15 \text{ ps}^{-1}$, $J_1 = 10 \text{ ps}^{-1}$, $J_2 = 12 \text{ ps}^{-1}$, $N_1 = 22$, $N_2 = 20$, $\alpha_1 = 250 \text{ ps}^{-1}$, $\alpha_2 = 10 \text{ ps}^{-1}$, $\Delta_1 = 20 \text{ ps}^{-1}$, $\Delta_2 = 10 \text{ ps}^{-1}$, $\Delta_3 = 22 \text{ ps}^{-1}$, $\Delta_4 = 12 \text{ ps}^{-1}$, and the temperature of the baths is 300 K.

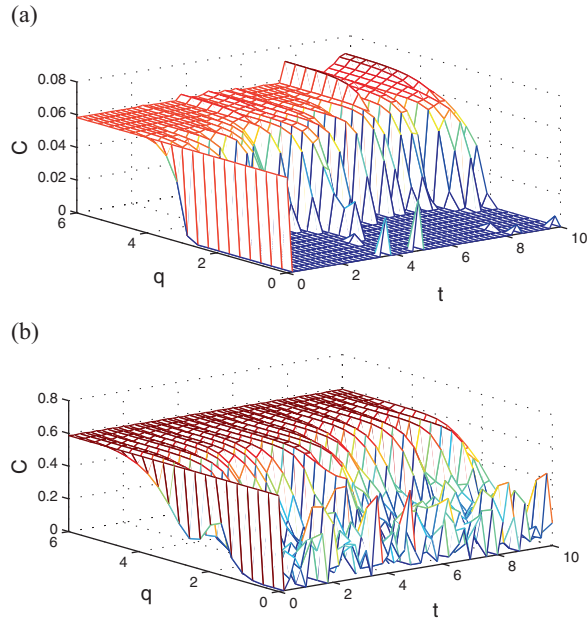


Fig. 2 (colour online). Change of the concurrence for the entanglement initial state. In (a), the initial state $\phi_d(0) = 0.5|1\rangle|3\rangle + 0.48|2\rangle|3\rangle + 0.49|1\rangle|4\rangle + 0.53|2\rangle|4\rangle$. In (b), the initial state $\phi_d(0) = 0.4|1\rangle|3\rangle + 0.2|2\rangle|3\rangle + 0.25|1\rangle|4\rangle + 0.86|2\rangle|4\rangle$. The parameters are: $\gamma_1 = 0.2 \text{ ps}^{-1}$, $\gamma_2 = 0.3 \text{ ps}^{-1}$, $\gamma_3 = 0.4 \text{ ps}^{-1}$, $\gamma_4 = 0.15 \text{ ps}^{-1}$, $J_1 = 10 \text{ ps}^{-1}$, $J_2 = 12 \text{ ps}^{-1}$, $N_1 = 22$, $N_2 = 20$, $\alpha_1 = 250 \text{ ps}^{-1}$, $\alpha_2 = 10 \text{ ps}^{-1}$, $\Delta_1 = 20 \text{ ps}^{-1}$, $\Delta_2 = 10 \text{ ps}^{-1}$, $\Delta_3 = 22 \text{ ps}^{-1}$, $\Delta_4 = 12 \text{ ps}^{-1}$, and the temperature of the baths is 300 K.

lution of the behaviour of the concurrence gradually changes from non-periodic evolution to periodic evolution and the amplitude of the concurrence becomes smaller. Finally, the fluctuation of the concurrence disappears when the coupling parameter q exceeds some value which depends on the initial state. For the initial states in Figure 2, the threshold value of the coupling parameter q is about 5. This means that we can obtain a steady entanglement by adjusting the coupling strength between the environments. This nature may provide promising prospect for the study of quantum information, the design of quantum devices, and entanglement preservation.

From the view of the non-equilibrium statistical physics, we may understand this novel result, namely the information transfer or the entropy transfer between the dimer system and the environments. Firstly, the coupling of the environments led to the exchange of information (entropy) between the environments, and this exchange has been gradually strengthened with in-

creasing q . Secondly, under the non-Markovian condition, the interaction between each single dimer system and its own spin bath not only makes the information (entropy) of the dimer system flow into the spin bath, but also arouses the feedback of the information (entropy) from the spin bath to the dimer system [4, 6, 8], showing sudden birth and sudden death phenomena. Finally, based on the above two reasons, there is a mutual influence between the information (entropy) of the two dimer systems, and this mutual influence strengthens the coherence of the dimer systems with increasing q . Especially, for the initial state of entanglement, a steady entanglement can be obtained when the parameter q exceeds some value depending on the initial state.

Further, we study an interesting special case, namely all the environments with zero temperature. Under this condition, all the spin baths will be in the ground states, and the ground states of the spin baths have the following forms:

$$\begin{aligned}
 |\psi_B^1(0)\rangle &= \left| \frac{N_1}{2}, -\frac{N_1}{2} \right\rangle \otimes \left| \frac{N_2}{2}, -\frac{N_2}{2} \right\rangle, \\
 &\text{for } q < q_0 \\
 |\psi_B^2(0)\rangle &= \left| \frac{N_1}{2}, -\frac{N_1}{2} \right\rangle \otimes \left| \frac{N_2}{2}, \frac{N_2}{2} \right\rangle, \\
 &\text{for } q > q_0, \alpha_1 > \alpha_2 \\
 |\psi_B^3(0)\rangle &= \left| \frac{N_1}{2}, \frac{N_1}{2} \right\rangle \otimes \left| \frac{N_2}{2}, -\frac{N_2}{2} \right\rangle, \\
 &\text{for } q > q_0, \alpha_1 < \alpha_2 \\
 q_0 &= 2 \min \left(\frac{\alpha_1}{N_2}, \frac{\alpha_2}{N_1} \right).
 \end{aligned} \tag{14}$$

In the case of degeneracy of the parameters, for example, $q_0 = q$ or $\alpha_1 = \alpha_2$, the ground state is the linear combination of $|\psi_B^1(0)\rangle$, $|\psi_B^2(0)\rangle$, and $|\psi_B^3(0)\rangle$.

Considering the reservoirs H_{B_i} commuting with H_d , $H_{d_1 B_i}$, and $H_{d_2 B_i}$, the states of the system will have the following form for the random initial condition $\phi_d(0) = a_{13}|1\rangle|3\rangle + a_{23}|2\rangle|3\rangle + a_{14}|1\rangle|4\rangle + a_{24}|2\rangle|4\rangle$ (satisfy $|a_{13}|^2 + |a_{23}|^2 + |a_{14}|^2 + |a_{24}|^2 = 1$):

$$|\psi_{\text{Total}}(t)\rangle = e^{-i\Theta t} \sum_{i=1,2;\ell=1,2,3}^{k=3,4} f_{ik}(t)|i\rangle|k\rangle \otimes |\psi_B^\ell(0)\rangle, \tag{15}$$

where $\Theta = \langle \psi_B^\ell(0) | \alpha_1 S_1^z + \alpha_2 S_2^z + q S_1^z S_2^z | \psi_B^\ell(0) \rangle$, and the index ℓ change according to the value of q_0 . The $f_{ik}(t)$ are given in the Appendix.

After we trace out the degrees of freedom of the spin bath, the dynamic evolution of the system is obtained. Using the definition of concurrence $C(\rho)$ [34], we can obtain the following expression about concurrence and further study the properties of the entanglement:

$$\begin{aligned} C(\text{Tr}_E(|\Psi_{\text{Total}}(t)\rangle\langle\Psi_{\text{Total}}(t)|)) \\ = 2|a_{13}a_{24} - a_{14}a_{23}|. \end{aligned} \quad (16)$$

From above equation, we know that the concurrence of the random initial states do not change with time under zero temperature. It can be understood from the view of the non-equilibrium statistical physics that the information or the entropy is transferred between the dimer system and the environments. For non-Markovian dynamics, the action of the environments do not make the information (entropy) of the dimer system flow into the environments, and the changing of the environments always feedbacks some information (entropy) to the dimer system. However, for zero temperature, because the environments do not change, there does not exist an information (entropy) transfer between the dimer system and the environments.

4. Conclusions

The possible relevance of the Fermi-spin environment to the entanglement has been investigated. For the initial state of entanglement or disentanglement, we show that the correlations between the spin baths with coupling strength parameter q impact on the evolution of the entanglement. Especially, for the entangling initial state, we can effectively depress the entanglement decoherence by increasing the value of the parameter q , and a steady entanglement can be obtained when q exceeds the critical value. This means that we can realize the preservation and control of entanglement by adjusting the coupling strength between the environments in the dimer system. The results of this paper may be practically useful for further study of the controllable quantum device, entanglement preservation [17–22], the quantum information of the biological systems, and more complex environmental models.

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Appendix

In this appendix, we did not only give the parameters A , D , G , and $f_{ik}(t)$ in (11) and (15), respectively, but also the main process about the parameters calculation method is been given. Considering the commutation relations of $H_{d'_1}$, $H_{d'_2}$, $\sum_{i=1,2} \alpha_i S_i^z$, and $q S_1^z S_2^z$, the main process has the following steps.

The eigenvalues of $H_{d'_1} + H_{d'_2}$:

$$\begin{aligned} \Delta_{1,p} &= \Delta_1 + z_{1,p}, \quad \Delta_{2,p} = \Delta_2 + z_{2,p}, \\ \Delta_{3,p} &= \Delta_3 + z_{3,p}, \quad \Delta_{4,p} = \Delta_4 + z_{4,p}, \end{aligned} \quad (p = 0, 1)$$

$$z_{1,0} = \langle \psi_B^\ell(0) | \frac{\gamma_1 S_1^z + \gamma_2 S_2^z}{2} | \psi_B^\ell(0) \rangle,$$

$$z_{2,0} = \langle \psi_B^\ell(0) | \frac{\gamma_2 S_2^z - \gamma_1 S_1^z}{2} | \psi_B^\ell(0) \rangle,$$

$$z_{3,0} = \langle \psi_B^\ell(0) | \frac{\gamma_3 S_1^z + \gamma_4 S_2^z}{2} | \psi_B^\ell(0) \rangle,$$

$$z_{4,0} = \langle \psi_B^\ell(0) | \frac{\gamma_4 S_2^z - \gamma_3 S_1^z}{2} | \psi_B^\ell(0) \rangle,$$

$$z_{1,1} = \frac{\gamma_1 m_1 + \gamma_2 m_2}{2}, \quad z_{2,1} = -\frac{\gamma_1 m_1 - \gamma_2 m_2}{2},$$

$$z_{3,1} = \frac{\gamma_3 m_1 + \gamma_4 m_2}{2}, \quad z_{4,1} = -\frac{\gamma_3 m_1 - \gamma_4 m_2}{2},$$

$$E_{1,p} = \Delta_{1,p} + \Delta_{3,p} + \sqrt{(\Delta_{2,p})^2 + J_1^2} + \sqrt{(\Delta_{4,p})^2 + J_2^2},$$

$$E_{2,p} = \Delta_{1,p} + \Delta_{3,p} + \sqrt{(\Delta_{2,p})^2 + J_1^2} - \sqrt{(\Delta_{4,p})^2 + J_2^2},$$

$$E_{3,p} = \Delta_{1,p} + \Delta_{3,p} - \sqrt{(\Delta_{2,p})^2 + J_1^2} + \sqrt{(\Delta_{4,p})^2 + J_2^2},$$

$$E_{4,p} = \Delta_{1,p} + \Delta_{3,p} - \sqrt{(\Delta_{2,p})^2 + J_1^2} - \sqrt{(\Delta_{4,p})^2 + J_2^2}.$$

The conversion factors between the states $|i\rangle|k\rangle$ ($i = 1, 2; k = 3, 4$) and the eigenvectors of $H_{d'_1} + H_{d'_2}$:

$$M_{1,p} = \frac{1}{4} \frac{J_1 J_2}{\sqrt{(\Delta_{2,p})^2 + J_1^2} \sqrt{(\Delta_{4,p})^2 + J_2^2}},$$

$$M_{2,p} = \frac{1}{4} \begin{bmatrix} 1 - \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \\ \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \end{bmatrix} \begin{bmatrix} J_2 \\ \sqrt{(\Delta_{4,p})^2 + J_2^2} \end{bmatrix},$$

$$M_{3,p} = \frac{1}{4} \begin{bmatrix} 1 + \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \\ \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \end{bmatrix} \begin{bmatrix} J_2 \\ \sqrt{(\Delta_{4,p})^2 + J_2^2} \end{bmatrix},$$

$$M_{4,p} = \frac{1}{4} \begin{bmatrix} J_1 \\ \sqrt{(\Delta_{2,p})^2 + J_1^2} \end{bmatrix} \begin{bmatrix} 1 - \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \\ \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \end{bmatrix},$$

$$\begin{aligned}
M_{5,p} &= \frac{1}{4} \left[\frac{J_1}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \right] \left[1 + \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \right], \\
M_{6,p} &= \frac{1}{4} \left[1 - \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \right] \left[1 - \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \right], \\
M_{7,p} &= \frac{1}{4} \left[1 + \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \right] \left[1 - \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \right], \\
M_{8,p} &= \frac{1}{4} \left[1 - \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \right] \left[1 + \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \right], \\
M_{9,p} &= \frac{1}{4} \left[1 + \frac{\Delta_{2,p}}{\sqrt{(\Delta_{2,p})^2 + J_1^2}} \right] \left[1 + \frac{\Delta_{4,p}}{\sqrt{(\Delta_{4,p})^2 + J_2^2}} \right], \\
Q_{1,p} &= \frac{\Delta_{2,p} + \sqrt{(\Delta_{2,p})^2 + J_1^2}}{J_1}, \\
Q_{2,p} &= \frac{\Delta_{2,p} - \sqrt{(\Delta_{2,p})^2 + J_1^2}}{J_1}, \\
Q_{3,p} &= \frac{\Delta_{4,p} + \sqrt{(\Delta_{4,p})^2 + J_2^2}}{J_2}, \\
Q_{4,p} &= \frac{\Delta_{4,p} - \sqrt{(\Delta_{4,p})^2 + J_2^2}}{J_2}.
\end{aligned}$$

The expression of the parameters $f_{ik}(t)$:

$$\begin{aligned}
R_1 &= [a_{13}M_{1,0} + a_{23}M_{2,0} + a_{14}M_{4,0} + a_{24}M_{6,0}], \\
R_2 &= [-a_{13}M_{1,0} - a_{23}M_{2,0} + a_{14}M_{5,0} + a_{24}M_{8,0}], \\
R_3 &= [-a_{13}M_{1,0} + a_{23}M_{3,0} - a_{14}M_{4,0} + a_{24}M_{7,0}],
\end{aligned}$$

$$\begin{aligned}
R_4 &= [a_{13}M_{1,0} - a_{23}M_{3,0} - a_{14}M_{5,0} + a_{24}M_{9,0}], \\
f_{13}(t) &= R_1 Q_{1,0} Q_{3,0} e^{-iE_{1,0}t} + R_2 Q_{1,0} Q_{4,0} e^{-iE_{2,0}t} \\
&\quad + R_3 Q_{2,0} Q_{3,0} e^{-iE_{3,0}t} + R_4 Q_{2,0} Q_{4,0} e^{-iE_{4,0}t}, \\
f_{23}(t) &= R_1 Q_{3,0} e^{-iE_{1,0}t} + R_2 Q_{4,0} e^{-iE_{2,0}t} \\
&\quad + R_3 Q_{3,0} e^{-iE_{3,0}t} + R_4 Q_{4,0} e^{-iE_{4,0}t}, \\
f_{14}(t) &= R_1 Q_{1,0} e^{-iE_{1,0}t} + R_2 Q_{1,0} e^{-iE_{2,0}t} \\
&\quad + R_3 Q_{2,0} e^{-iE_{3,0}t} + R_4 Q_{2,0} e^{-iE_{4,0}t}, \\
f_{24}(t) &= R_1 e^{-iE_{1,0}t} + R_2 e^{-iE_{2,0}t} \\
&\quad + R_3 e^{-iE_{3,0}t} + R_4 e^{-iE_{4,0}t}.
\end{aligned}$$

The expression of the parameters A , D , and G :

$$\begin{aligned}
(A)^\dagger &= (\langle 3|\langle 1| \quad \langle 3|\langle 2| \quad \langle 4|\langle 1| \quad \langle 4|\langle 2|) \\
G &= \begin{pmatrix} a_{13}a_{13}^* & a_{23}a_{13}^* & a_{14}a_{13}^* & a_{24}a_{13}^* \\ a_{13}a_{23}^* & a_{23}a_{23}^* & a_{14}a_{23}^* & a_{24}a_{23}^* \\ a_{13}a_{14}^* & a_{23}a_{14}^* & a_{14}a_{14}^* & a_{24}a_{14}^* \\ a_{13}a_{24}^* & a_{23}a_{24}^* & a_{14}a_{24}^* & a_{24}a_{24}^* \end{pmatrix}, \\
M &= \begin{pmatrix} M_{1,1} & -M_{1,1} & -M_{1,1} & M_{1,1} \\ M_{2,1} & -M_{2,1} & M_{3,1} & -M_{4,1} \\ M_{4,1} & M_{5,1} & -M_{4,1} & -M_{5,1} \\ M_{6,1} & M_{8,1} & M_{7,1} & M_{9,1} \end{pmatrix}, \\
B &= \begin{pmatrix} e^{-iE_{1,1}t} & 0 & 0 & 0 \\ 0 & e^{-iE_{2,1}t} & 0 & 0 \\ 0 & 0 & e^{-iE_{3,1}t} & 0 \\ 0 & 0 & 0 & e^{-iE_{4,1}t} \end{pmatrix}, \\
Q &= \begin{pmatrix} Q_{1,1}Q_{3,1} & Q_{3,1} & Q_{1,1} & 1 \\ Q_{1,1}Q_{4,1} & Q_{4,1} & Q_{1,1} & 1 \\ Q_{2,1}Q_{3,1} & Q_{3,1} & Q_{2,1} & 1 \\ Q_{2,1}Q_{4,1} & Q_{4,1} & Q_{2,1} & 1 \end{pmatrix}, \\
D &= (Q)^\dagger B^\dagger (M)^\dagger GMBQ.
\end{aligned}$$

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