Lubrication Performances of Short Journal Bearings Operating with Non-Newtonian Ferrofluids

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The lubrication performances of short journal bearings operating with non-Newtonian ferrofluids have been investigated in the present study. Based upon the ferrofluid model of Shliomis and the micro-continuum theory of Stokes, a two-dimensional modified Reynolds equation is derived by taking into account the effects of rotation of ferromagnetic particles and the effects of non-Newtonian properties. As an application, the short-bearing approximation is illustrated. Comparing with the conventional non-ferrofluid case, the short journal bearings with ferrofluids in the presence of magnetic fields result in a higher load capacity. Comparing with the Newtonian ferrofluid case, the non-Newtonian effects of couple stresses provide an enhancement in the load capacity, as well as a reduction in the friction parameter. The inclusion of non-Newtonian couple stresses signifies an improvement in performance characteristics of ferrofluid journal bearings.

Key words: Ferrofluids; Non-Newtonian Effects; Journal Bearings; Reynolds Equation; Steady-State Performances.

1. Introduction

The application of ferrofluids in lubrication fields has received great attention in recent years. A ferrofluid is a stable colloidal suspension of small magnetic particles in a base fluid. When a magnetic field is applied to the magnetic fluid, each ferromagnetic particle experiences a force that depends on the magnetization of the particles and on the strength of the applied magnetic fields. Using ferrofluids as lubricants, several studies of journal bearing characteristics are observed in the literature. For example, Tipei [1] presented that ferromagnetic lubricants may improve substantially the performance of short bearings operating under low loads and/or at low speeds. Sorge [2] found that the load capacities of magnetic bearings are better than conventional bearings when the magnetic field and saturation magnetization are high, the rotation speed is low, and the relative clearance is large. Zhang [3] suggested that the position of the oil film should be located with the positive magnetic force region. It is found that the main feature of the magnetic force is the ability to change the size of the cavitation region and all the static characteristics. However, the effects of rotation of ferromagnetic particles are neglected in these studies.

According to the ferrofluid model of Shliomis [4], the orienting magnetic field impedes rotation of the particles and increases the effective viscosity. Therefore, the effects of rotation of ferromagnetic particles should be included.

On the other hand, the experimental evidence by Oliver and Shahidullah [5] and Oliver [6] has show that the lubricant blended with small amounts of additives can reduce the wear of friction material. Several rheological models such as power law fluids, Herschel–Bulkley fluids, and Rabinowitsch fluids can be observed in the literature. In addition, a micro-continuum theory taking into account the intrinsic motion of material constituents has been proposed by Stokes [7] to describe these kinds of non-Newtonian lubricants. It allows for polar effects such as the presence of couple stresses, body couples, and non-symmetric tensors. The couple stress fluid model of Stokes micro-continuum theory is intended to account for the particle-size effects and is important for applications of fluids with additives. Since the material constant responsible for couple stress fluids could also be determined by the experimental procedure as described by Stokes [7], this micro-continuum theory is widely applied in the study of lubrication problems. On the ground of this couple stress fluid model, several researches of journal bearing characteristics are presented such as the short bearings by Lin [8], the finite bearings by Lin [9], and the long bearings by Liao et al. [10]. It is found that the effects of non-Newtonian couple stresses show apparent influences on the lubrication performances of journal bearings. Based upon the above contributions, the ferrofluids with non-Newtonian behaviour would arise in the journal bearings. Therefore, a further study is needed.

Based upon the ferrofluid model of Shliomis [4] and the micro-continuum theory of Stokes [7], the lubrication performances of short journal bearings operating with non-Newtonian ferrofluids are concerned in this study. A two-dimensional modified Reynolds equation is derived by taking into account the effects of rotation of ferromagnetic particles and the effects of non-Newtonian properties. As an application, the short-bearing approximation is illustrated. Expressions for bearing characteristics are derived. Comparing with the bearing with a Newtonian non-ferrofluid, the load capacity and the friction parameter are presented for various values of the volume concentration, the Langevin parameter, and the couple stress parameter.

2. Analysis

The physical configuration of a journal bearing is shown in Figure 1. The x-direction defines the direction perpendicular to the line of centers of the bearing and the journal, the y-direction corresponds to the direction across the film, and the z-direction defines the direction of the length of the bearing. The journal of radius R is rotating with a uniform tangential velocity U within a metal bearing housing. The lubricant is taken to be an incompressible non-Newtonian couple stress ferrofluid. A uniform magnetic field $\vec{H} = (0, H_0, 0)$ is applied across the fluid film. In experiment, the uniformity of the magnetic field is produced by a Helmholtz coil [11, 12]. Assume that the inertia forces are negligible, the body forces and body couples are absent, and the second derivative of the internal angular momentum is neglected. In addition, the ferrofluid viscosity is also assumed to be defined for the dilute regime.

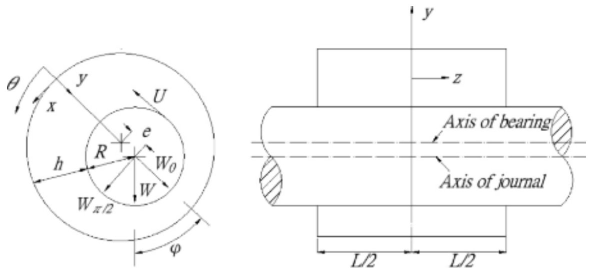

Fig. 1. Physical configuration of a journal bearing lubricated with a non-Newtonian ferrofluid.

According to the ferrofluid model of Shliomis [4] and the micro-continuum theory of Stokes [7], the momentum equations governing the fluid flow can be expressed as

$$-\nabla p + \mu(1+\tau)\nabla^2\vec{V} + \mu_0\left(\vec{M}\cdot\nabla\right)\vec{H} - \eta\nabla^4\vec{V} = 0\,, \tag{1}$$

where p is the pressure, μ the viscosity of the suspension, μ_0 the permeability of free space, η a material constant responsible for the couple stress fluid property, $\vec{M}$ the magnetization vector, and $\vec{V} = (u, v, w)$ the fluid velocity.

Under the usual lubrication theory, the pressure gradient in the y-direction is neglected. Then the momentum equations and the incompressible continuity equation expressed in Cartesian coordinates reduce to

$$\frac{\partial p}{\partial x} = \mu(1+\tau)\frac{\partial^2 u}{\partial y^2} - \eta\frac{\partial^4 u}{\partial y^4}\,, \tag{2}$$

$$\frac{\partial p}{\partial y} = 0\,, \tag{3}$$

$$\frac{\partial p}{\partial z} = \mu(1+\tau)\frac{\partial^2 w}{\partial y^2} - \eta\frac{\partial^4 w}{\partial y^4}\,, \tag{4}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0\,, \tag{5}$$

where

$$\tau = \frac{3}{2}\phi\frac{\xi - \tanh\xi}{\xi + \tanh\xi}\,, \tag{6}$$

$$\mu = \mu_{\mathrm{L}}\left(1 + \frac{5}{2}\phi\right)\,. \tag{7}$$

In these equations, ξ is the Langevin parameter, ϕ the volume concentration of the particles, and μ_L the viscosity of the main liquid. The velocity boundary conditions for the lubricant flow are the no slip conditions and the non couple-stress conditions at the bearing surfaces:

$$u(x,0,z) = v(x,0,z) = w(x,0,z) = 0\,, \tag{8}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=0} = 0\,, \tag{9}$$

$$u(x,h,z) = U,\;\; v(x,h,z) = w(x,h,z) = 0\,, \tag{10}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=h} = \left.\frac{\partial^2 w}{\partial y^2}\right|_{y=h} = 0\,. \tag{11}$$

Integrating (2) and (4), respectively, and applying the related boundary conditions, the velocity components are derived:

$$\begin{aligned} u = {} & \frac{y}{h}U + \frac{A}{2\mu_L(1+\tau)} \cdot \frac{\partial p}{\partial x} \\ & \cdot \left\{ y(y-h) + 2l^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)} \right] \right\}, \end{aligned} \tag{12}$$

$$\begin{aligned} w = {} & \frac{A}{2\mu_L(1+\tau)} \cdot \frac{\partial p}{\partial z} \\ & \cdot \left\{ y(y-h) + 2l^2 \left[1 - \frac{\cosh\left(\frac{2y-h}{2l}\right)}{\cosh\left(\frac{h}{2l}\right)} \right] \right\}, \end{aligned} \tag{13}$$

where $l^2 = l_c^2 A/(1+\tau)$ with $l_c = \sqrt{\eta/\mu_L}$ being the couple stress parameter and $A = 1/(1+5\phi/2)$. Substituting the expressions of the velocity components into the continuity equation (5) and integrating with respect to y, a modified Reynolds equation for journal bearings operating with non-Newtonian ferrofluids is derived:

$$\begin{aligned} & \frac{\partial}{\partial x}\left[f(h,l_c,\phi,\xi)\frac{\partial p}{\partial x} \right] + \frac{\partial}{\partial z}\left[f(h,l_c,\phi,\xi)\frac{\partial p}{\partial z} \right] \\ & = 6\mu_L \frac{1}{A}(1+\tau)U\frac{\partial h}{\partial x}\,, \end{aligned} \tag{14}$$

where

$$\begin{aligned} f(h,l_c,\phi,\xi) = {} & h^3 - 12\frac{l_c^2 A}{1+\tau}h + 24\frac{l_c^2 A}{1+\tau}\frac{l_c\sqrt{A}}{\sqrt{1+\tau}} \\ & \cdot \tanh\left(\frac{\sqrt{1+\tau}}{2l_c\sqrt{A}}h \right). \end{aligned} \tag{15}$$

Introduce the dimensionless variables and parameters as follows:

$$p^* = \frac{pC^2}{\mu_L UR}\,,\;\; \theta = \frac{x}{R}\,,\;\; z^* = \frac{z}{L}\,,\;\; \lambda = \frac{L}{2R}\,, \tag{16}$$

$$h^* = \frac{h}{C} = 1 + \varepsilon\cos\theta\,,\;\; \varepsilon = \frac{e}{C}\,,\;\; l_c^* = \frac{l_c}{C}\,. \tag{17}$$

In these definitions, L, C, and e are the bearing length, the radial clearance, and the eccentricity of the journal center, respectively. As a consequence, the modified Reynolds equation can be written in a dimensionless form as

$$\begin{aligned} & \frac{\partial}{\partial\theta}\left[f^*(h^*,l_c^*,\phi,\xi)\frac{\partial p^*}{\partial\theta} \right] + \frac{1}{4\lambda^2}\frac{\partial}{\partial z^*} \\ & \cdot \left[f^*(h^*,l_c^*,\phi,\xi)\frac{\partial p^*}{\partial z^*} \right] = 6\frac{1}{A}(1+\tau)\frac{\partial h^*}{\partial\theta}\,, \end{aligned} \tag{18}$$

where

$$\begin{aligned} f^*(h^*,l_c^*,\phi,\xi) = {} & h^{*^3} - 12\frac{l_c^{*^2}A}{1+\tau}h^* + 24\frac{l_c^{*^2}A}{1+\tau} \\ & \cdot \frac{l_c^*\sqrt{A}}{\sqrt{1+\tau}}\tanh\left(\frac{\sqrt{1+\tau}}{2l_c^*\sqrt{A}}h^* \right). \end{aligned} \tag{19}$$

3. Bearing Characteristics

The modified Reynolds equation derived can be applied to the study of two-dimensional problems. In order to simplify the problem, a short bearing approximation ($\lambda^2 \ll 1$) is assumed, for which the circumferential variation of pressure can be neglected as compared to the axial variations. Then the two-dimensional dimensionless modified Reynolds equation reduces to

$$\frac{\partial}{\partial z^*}\left[f^*(h^*,l_c^*,\phi,\xi)\frac{\partial p^*}{\partial z^*} \right] = 24\lambda^2\frac{1}{A}(1+\tau)\frac{\partial h^*}{\partial\theta}\,. \tag{20}$$

The pressure boundary conditions are

$$\frac{\partial p^*}{\partial z^*} = 0 \;\text{ at }\; z^* = 0\,, \tag{21}$$

$$p^* = 0 \;\text{ at }\; z^* = \pm\frac{1}{2}\,. \tag{22}$$

Integrating (20) by applying the pressure boundary conditions, one can obtain the dimensionless film pressure

$$p^* = 12\lambda^2\frac{1}{A}(1+\tau)\left(\frac{1}{4} - z^{*^2}\right)\frac{\varepsilon\sin\theta}{f^*(h^*,l_c^*,\phi,\xi)}\,. \tag{23}$$

The load capacity can be obtained by integrating the film pressure acting on the journal surfaces. The dimensionless load components along and perpendicular to the line of the center, W_0^* and $W_{\pi/2}^*$, are given by

$$W_0^* = W^* \cos\varphi = -2\int_0^{\pi}\int_0^{1/2} p^* \cos\theta \, dz^* \, d\theta \,, \qquad (24)$$

$$W_{\pi/2}^* = W^* \sin\varphi = 2\int_0^{\pi}\int_0^{1/2} p^* \sin\theta \, dz^* \, d\theta \,, \qquad (25)$$

where the attitude angle can be calculated from $\varphi = \tan^{-1}(W_{\pi/2}^*/W_0^*)$. As a result, the dimensionless load capacity W^* is obtained:

$$W^* = \sqrt{W_0^{*2} + W_{\pi/2}^{*2}} \,. \qquad (26)$$

The friction force can be evaluated by integrating the shear stress τ_s at the journal surface that is

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=h} - \eta \frac{\partial^3 u}{\partial y^3}\bigg|_{y=h} \,. \qquad (27)$$

Therefore, the dimensionless frictional force at the journal surface is evaluated to

$$F^* = 2\int_0^{2\pi}\int_0^{1/2} \tau_s \, dz^* \, d\theta \,. \qquad (28)$$

As a result, the friction parameter f_p can be obtained:

$$f_p = \frac{F}{W}\frac{R}{C} = \frac{F^*}{W^*} \,. \qquad (29)$$

4. Results and Discussion

According to the above analysis, the lubrication performances of short journal bearings with non-Newtonian ferrofluids are characterized by the volume concentration ϕ, the Langevin parameter ξ, and the couple stress parameter l_c^*.

Case 1. for $l_c^* \neq 0$, $\phi = 0$, and $\xi = 0$, the non-Newtonian non-ferrofluid case without magnetic fields. The derived modified Reynolds equation (20) reduces to the equation of short bearings with non-Newtonian fluids derived by Lin [8],

$$\frac{\partial}{\partial z^*}\left[f^*(h^*, l_c^*, 0, 0)\frac{\partial p^*}{\partial z^*}\right] = 24\lambda^2 \frac{\partial h^*}{\partial \theta} \,. \qquad (30)$$

Case 2. for $l_c^* = 0$, $\phi = 0$, and $\xi = 0$, the Newtonian non-ferrofluid case without magnetic fields. The derived modified Reynolds equation (14) reduces to the Reynolds equation of short bearings with Newtonian fluids derived by Hamrock [13],

$$\frac{\partial}{\partial z}\left[h^3 \frac{\partial p}{\partial z}\right] = 6\mu_0 U \frac{\partial h}{\partial x} \,. \qquad (31)$$

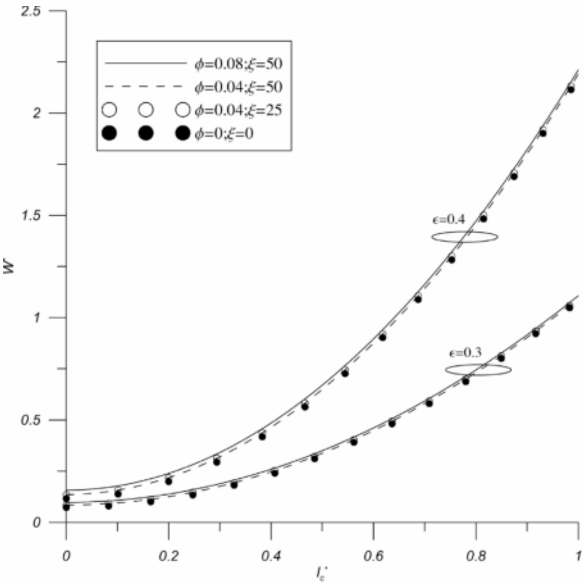

Fig. 2. Load capacity W^* versus the couple stress parameter l_c^* with various ϕ and ξ for $\varepsilon = 0.3$ and $\varepsilon = 0.4$.

In the present study, bearing characteristics are presented with different values of l_c^*, ϕ, and ξ. Figure 2 displays the dimensionless load capacity W^* versus the couple stress parameter l_c^* with various values of ϕ and for $\varepsilon = 0.3$ and $\varepsilon = 0.4$. The load capacity is observed to increase with increasing values of the couple stress parameter. For the bearing operating at eccentricity ratio $\varepsilon = 0.4$, the bearing with a ferrofluid ($\phi = 0.04$) in the presence of a magnetic field ($\xi = 25$) provides an increase in the load capacity as compared to the non-ferrofluid case ($\phi = 0$) without magnetic fields ($\xi = 0$). Increasing values of ϕ, ξ, and ε increases further the increments of load. Figure 3 describes the dimensionless load capacity W^* versus ε with various values of l_c^*, ϕ, and ξ. The load capacity is seen to increase with increasing values of ε. It is also observed that the bearing lubricated with a Newtonian ferrofluid without magnetic fields ($l_c^* = 0$, $\phi = 0.08$, $\xi = 0$) and with magnetic fields ($l_c^* = 0$, $\phi = 0.08$, $\xi = 50$) provides higher values of the load capacity than the bearing lubricated with Newtonian non-ferrofluids in absence of magnetic fields ($l_c^* = 0$, $\phi = 0$, $\xi = 0$). When the effects of non-Newtonian couple

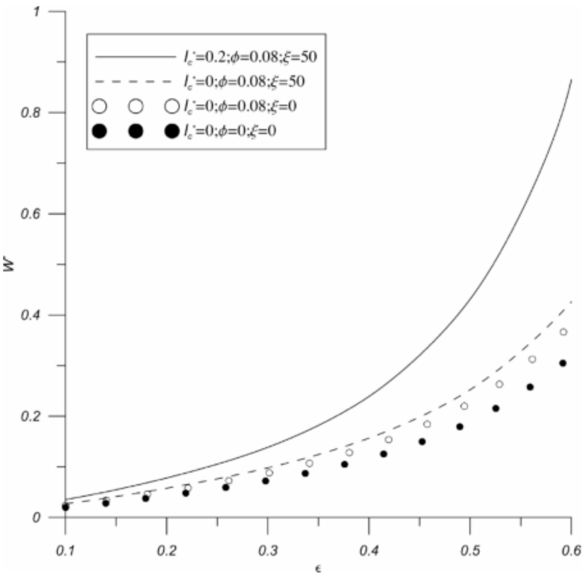

Fig. 3. Load capacity W^* versus ε with various ϕ and ξ for different couple stress parameter l_c^*.

stresses are considered ($l_c^* = 0.2$, $\phi = 0.08$, $\xi = 50$), further higher values of the load capacity are predicted for the journal bearing. Figure 4 shows the dimensionless load capacity W^* versus the Langevin parameter ξ with various ϕ and l_c^* for $\varepsilon = 0.4$. Comparing with the non-ferrofluid ($\phi = 0$), Newtonian ($l_c^* = 0$) case, the Newtonian ferrofluid-lubricated bearing ($\phi = 0.04$, $l_c^* = 0$) is found to yield high values of the load in the presence of magnetic fields. Increasing values of the volume concentration and the couple stress parameter ($\phi = 0.04$, $l_c^* = 0.1$; $\phi = 0.08$, $l_c^* = 0.2$), further increments of the load capacity are observed especially for the bearing operating under a higher value of the Langevin parameter ξ. Although the load capacity increases slightly with heavy magnetic circuits around a bearing, it is valuable for the slight increase by the magnetic fields. Because the ferrofluids as lubricants with the magnetic fields have various advantages such as long life, silent operation, high precision, and reduced wear. In application, these advantages are employed to the framework of mechanical (e. g. seal and bearings) and electromechnical (e. g. sensors and loudspeakers).

Figure 5 presents the friction parameter f_p versus the couple stress parameter l_c^* with various ϕ and ξ for $\varepsilon = 0.3$ and $\varepsilon = 0.4$. It shows that the friction parameter of the journal bearing decreases with increasing values of the couple stress parameter. It is also observed that the bearing with ferrofluids in the presence of magnetic field ($\phi = 0.04$, $\xi = 25$) under small values of l_c^* results in a smaller friction parameter as compared to the non-ferrofluid case without magnetic fields ($\phi = 0$, $\xi = 0$); but, reversed trends are obtained for the ferrofluid bearing with larger values of l_c^*. Since the friction parameter is calculated by $f_p = F^*/W^*$, smaller

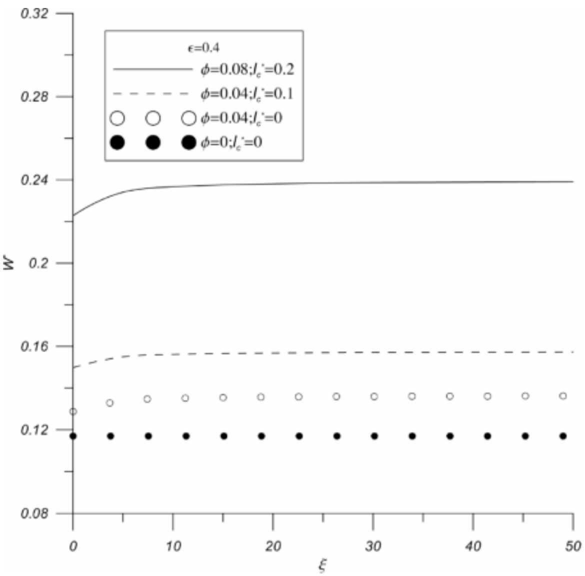

Fig. 4. Load capacity W^* versus the Langevin parameter ξ with various ϕ and l_c^* for $\varepsilon = 0.4$.

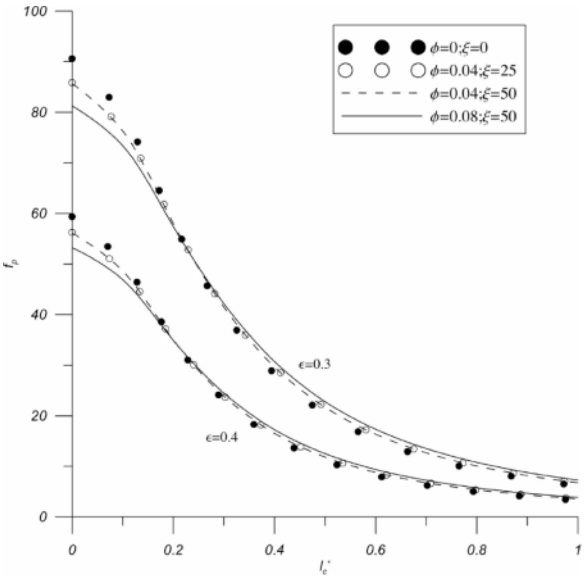

Fig. 5. Friction parameter f_p versus the couple stress parameter l_c^* with various ϕ and ξ for $\varepsilon = 0.3$ and $\varepsilon = 0.4$.

increments of F^* together with larger increments of W^* may result in a decrease in friction parameter. For example, the values of the friction force and the load capacity are

$$\begin{aligned}
&F^* = 6.6290\,,\ W^* = 0.0836\,,\ f_p = 79.3345\,,\\
&\text{for } \varepsilon = 0.3\,,\ l_c^* = 0.1\,,\ \phi = 0\,,\ \xi = 0\,;\\
&F^* = 7.9543\,,\ W^* = 0.1084\,,\ f_p = 73.3864\,,\\
&\text{for } \varepsilon = 0.3\,,\ l_c^* = 0.1\,,\ \phi = 0.08\,,\ \xi = 50\,;\\
&F^* = 6.6786\,,\ W^* = 0.4364\,,\ f_p = 15.3030\,,\\
&\text{for } \varepsilon = 0.3\,,\ l_c^* = 0.6\,,\ \phi = 0\,,\ \xi = 0\,;\\
&F^* = 8.0020\,,\ W^* = 0.4611\,,\ f_p = 17.3539\,,\\
&\text{for } \varepsilon = 0.3\,,\ l_c^* = 0.6\,,\ \phi = 0.08\,,\ \xi = 50\,.
\end{aligned}$$

Therefore, the ferrofluid bearing with applied magnetic fields under small couple stress parameters results in a smaller friction parameter as compared to the non-ferrofluid case without magnetic fields. Generally speaking, the non-Newtonian influences of couple stresses decrease the friction parameter of the journal bearing lubricated with ferrofluids in the presence of magnetic fields. Figure 6 describes the friction parameter f_p versus the Langevin parameter ξ with various ϕ and l_c^* for $\varepsilon = 0.4$. Comparing with the non-ferrofluid ($\phi = 0$), Newtonian ($l_c^* = 0$) case, the Newtonian ferrofluid-lubricated bearing ($\phi = 0.04$, $l_c^* = 0$) in the presence of magnetic fields ($\xi \neq 0$) is found to give a lower friction parameter. Increasing values of the volume concentration and the couple stress parameter ($\phi = 0.04$, $l_c^* = 0.1$; $\phi = 0.08$, $l_c^* = 0.2$), the effects of non-Newtonian ferrofluids are observed to provide a further reduction of the friction parameter. On the whole, the short journal bearings lubricated with non-Newtonian ferrofluids provide better lubrication performances than the conventional Newtonian non-ferrofluid bearings.

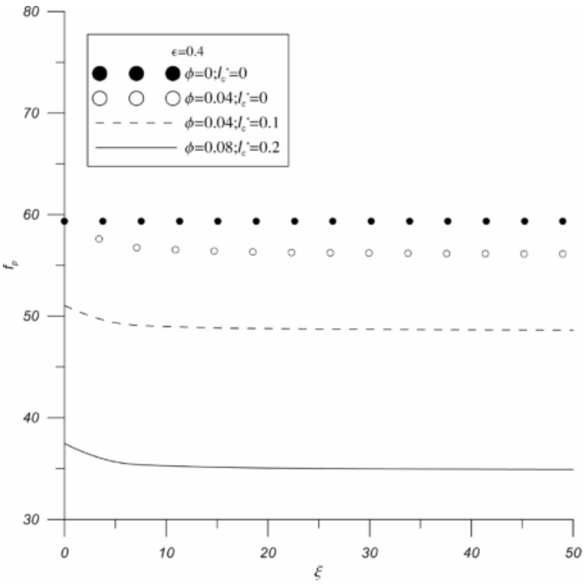

Fig. 6. Friction parameter f_p versus the Langevin parameter ξ with various ϕ and l_c^* for $\varepsilon = 0.4$.

5. Conclusions

On the basis of the Shliomis ferrofluid model and the Stokes micro-continuum theory, a two-dimensional modified Reynolds equation is derived by considering the rotational effects of ferromagnetic particles and the non-Newtonian effects of couple stresses.

Comparing with the conventional non-ferrofluid case, the short journal bearings with ferrofluids in the presence of magnetic fields result in a higher load capacity. Comparing with the Newtonian ferrofluid case, the non-Newtonian effects of couple stresses provide an enhancement in the load capacity, as well as a reduction in the friction parameter. Performance characteristics of ferrofluid journal bearings are improved when non-Newtonian influences of couple stresses are included.

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