

# Thermal Magnetoresistance, Magnetorelectivity, and Cyclotron Energy Loss Spectrum Effects Calculated for Electrons in Metals

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The thermal magnetoresistance, optical magnetorelectance, and extremal properties of the energy loss spectrum connected with the cyclotron resonance effect are approached on a theoretical basis. The formalism, which applies the idea of two kinds of relaxation times active for the electron magnetotransport in a metal, gives – at least qualitatively – a satisfactory agreement with the observed data in all examined cases.

*Key words:* Electron Thermo-Magnetoresistance; Optical Reflection in the Magnetic Field; Energy Loss Spectrum in the Cyclotron Resonance Effect; Metals.

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## 1. Introduction

The effect of the magnetic field on the electron transport in metals has been examined both experimentally and theoretically in many aspects and on many occasions. Perhaps the most widely investigated effect was the change of the electric resistance due to the change of the magnetic field. Experimentally the problem was approached a time ago [1] and found numerous continuations [2]. Theoretically its explanation was believed to be represented in [3].

In fact, numerous discrepancies between the expectations of theory and the data of experiment led some authors to conclusion that none of the magnetic field dependencies of the diagonal elements of the electrical or thermal magnetoresistivity tensors predicted by a semiclassical theory of the electron magnetotransport in metals are borne out by experiment [4].

In [5, 6], we tried to demonstrate that this situation, at least concerning the magnetoresistance of metals, can be improved if two separate relaxation times of the electron transport are considered. In this case, the tensor of the magnetoresistance is a sum of two tensors, one of them having the relaxation time  $\tau_{el}$ , the other one having the relaxation time  $\tau_{magn}$ :

$$\bar{\rho}_{el} = \frac{m}{n_s e^2 \tau_{el}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1)$$

$$\bar{\rho}_{magn} = \frac{m}{n_s e^2 \tau_{magn}} \begin{pmatrix} 1 & -\xi & 0 \\ \xi & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad (2)$$

$\tau_{el}$  is responsible for the action of the electric field on the metal, whereas  $\tau_{magn}$  is taking into account the effect of the magnetic field;  $n_s$  is the concentration of the electron carriers. Therefore the effective tensor for magnetoresistance becomes a sum of (1) and (2):

$$\begin{aligned} \bar{\rho}_{eff} = \bar{\rho}_{el} + \bar{\rho}_{magn} &= \frac{m}{n_s e^2 \tau_{eff}} \begin{pmatrix} 1 & -\xi \frac{\tau_{eff}}{\tau_{magn}} & 0 \\ \xi \frac{\tau_{eff}}{\tau_{magn}} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\ &= \frac{m}{n_s e^2 \tau_{eff}} \begin{pmatrix} 1 & -\xi' & 0 \\ \xi' & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \quad (3)$$

In calculating (3), we applied the formula

$$\frac{1}{\tau_{eff}} = \frac{1}{\tau_{el}} + \frac{1}{\tau_{magn}} \quad (4)$$

which is a consequence of the Matthiessen's rule combining  $\tau_{el}$  and  $\tau_{magn}$ . In the last step of (3), the substitution

$$\xi' = \xi \frac{\tau_{eff}}{\tau_{magn}} \quad (5)$$

has been done. This holds because the parameter entering (2) and (3) is

$$\xi = \frac{eB}{mc} \tau_{\text{magn}} = \Omega_0 \tau_{\text{magn}}. \quad (6)$$

A characteristic property of  $\xi$  in (6) is that it is a constant number independent of the induction  $B$ , so in effect  $\tau_{\text{magn}}$  becomes inversely proportional to  $B$  [5, 6].

In the developed formalism, we assumed that the electrons of a metal are considered as collected in a single band of states and these states are of a nearly-free electron character. Moreover, the magnetic field acting on the electron ensemble has a constant direction, say along axis  $z$  [7].

The change  $\Delta\rho$  of the electric resistance due to the magnetic field referred to a diagonal term of (1) calculated at  $B = 0$  becomes

$$\frac{\Delta\rho}{\rho(0)} = \frac{\tau_{\text{el}}}{\tau_{\text{magn}}}. \quad (7)$$

Since

$$\frac{1}{\tau_{\text{magn}}} = \frac{\Omega_0}{\xi}, \quad (8)$$

we have

$$\Omega_0 \sim B, \quad (9)$$

and  $\xi$  is a constant parameter calculated in [5]. Evidently, we obtain a systematic increase of the expression in (7) with an increase of  $B$ .

A comparison between the experiment and theory on the basis of (7) has been done for numerous experimental situations [5, 6, 8, 9]. Because  $\tau_{\text{el}}$  in (7) is experimentally a well-accessible parameter strongly dependent on the metal temperature, but the temperature dependence does not apply to  $\tau_{\text{magn}}$ , the dependence of magnetoresistance in (7) on the temperature change is easy to obtain from the corresponding experimental temperature dependence of  $\tau_{\text{el}}$ ; see [9].

The aim of the present paper is to extend a comparison between the experiment and theory on the thermal magnetotransport of electrons and the magnetorefectivity effect in metals. In spite of an old experimental basis of these phenomena, an adequate theoretical approach seemed here to be lacking. Another kind of the examined effects concerns the energy loss spectra obtained in course of the cyclotron resonance phenomenon due to the magnetic field. The applied formalism allowed us to approach the cyclotron resonance

frequency on a purely theoretical footing, i.e. without any use of the empirical parameters. A comparison of the calculated data with experiment can give an estimate of the effective mass of the metal electrons.

## 2. Thermal Magnetoresistance Effect Calculated and Compared with Experiment

The change of the electronic thermal resistance of a metal due to the magnetic field is strictly connected with the electric magnetoresistance of that metal. Similarly to the magnetoresistance of the electron transport with no thermal effects considered, an experimental background for the change of the electronic thermal resistance upon the action of the magnetic field became quite old [10, 11]. But, at the same time, a satisfactory computational basis of the phenomenon seemed to be lacking. In fact, having an approach to the electric magnetoresistance, an insight into the thermal electron resistance can be attained theoretically in a simple way.

A well-known relation between the thermal electron conductivity  $\lambda(0)$  and the electric conductivity  $\sigma(0)$  existing in the absence of the magnetic field ( $B = 0$ ; see e.g. [12]) gives

$$\lambda(0) = L_e T \sigma(0). \quad (10)$$

Here  $L_e$  is the Lorenz number for the electronic conductivity which may differ from the normal (free-electron) value  $L_n$  [12],  $T$  is the absolute temperature. We have omitted a small component  $\lambda_l$  of  $\lambda(0)$  which remains unaffected by the magnetic field [12].

Relation (10) represents the essence of the Wiedemann–Franz law; it can be transformed, for the sake of convenience, into the formula between the thermal resistivity

$$k = \frac{1}{\lambda} \quad (11)$$

and the electric resistance

$$\rho = \frac{1}{\sigma}. \quad (12)$$

In the absence of a magnetic field  $B$ , we have [12]

$$\frac{1}{k(0)} = L_e T \frac{1}{\rho(0)} \quad (13)$$

or

$$\rho(0) = L_e T k(0). \quad (13a)$$

The results obtained experimentally by Grueneisen and Erfling [13] (see also [12]) show that the changes  $\Delta\rho$  and  $\Delta k$  due to the changes of the magnetic field satisfy an equation much similar to (13a):

$$\Delta\rho \cong L_n T \Delta k. \quad (14)$$

By dividing (14) by (13a), we obtain

$$\frac{\Delta\rho}{\rho(0)} \cong \frac{L_n}{L_e} \frac{\Delta k}{k(0)}; \quad (15)$$

here the ratio  $L_n/L_e$  is an approximately constant term usually not much different than unity; see [12, 14].

Therefore, the change of the thermal resistivity in the magnetic field referred to the resistivity in the absence of that field is approximately equal to a similar change of the electric resistance leading to the magnetoresistance effect. The relation (15) is applied below in calculating the dependence of the ratio  $\Delta k/k(0)$  on  $B$  and  $T$ .

The property of proportionality of the thermal magnetoresistance effect to the electric magnetoresistance (see (15)) implies immediately a linear dependence

$$\frac{\Delta k}{k(0)} \sim B \quad (16)$$

obtained before [5, 6, 8] for the magnetoresistance alone.

This behaviour is in fact observed in numerous metal cases examined experimentally (Zn, Pb, Cd, Tl, In, Ag, Sn, Ga in [10], Zn in [15], Be in [12, 16]. For beryllium, instead of the resistance changes the effect of the magnetic field on the corresponding kinds of the metal conductivity were examined). Similar observations could be done also for the metal samples in which the size effects enter into play, for example a thin rod of sodium [11, 17]. Nevertheless, for the metallic potassium [4] and tungsten [12, 17] rather a parabolic increase of the thermal magnetoresistance with the magnetic field than a linear behaviour is observed. This discrepancy cannot be explained in the framework of the present theory. However, for potassium metal a strictly linear dependence of the longitudinal thermal magnetoresistance on  $B$  is experimentally obtained [4].

It should be noted that the reported results concerned the magnetic field larger than the critical field, so the metals being submitted to low temperatures remained in their normal state. The dependence of  $L_e$  on

$B$  in (15) has been also examined on the experimental basis (see e.g. [12, 18]) and this shows rather limited changes of that parameter with  $B$ . A very convincing experimental illustration of the property of proportionality existent between the electric and thermal magnetoresistance is provided by the measurements done on the both resistance kinds for the case of zinc [15]. These data plotted for different angles of the magnetic field taken for a constant  $B$ , exhibit an evident parallel behaviour.

A separate problem is an estimate of the slope of increase of the thermal magnetoresistance with an increase of  $B$ . This slope should be a temperature-dependent quantity because the resistance  $\rho_{el}(B=0)$  and the relaxation time  $\tau_{el}$  entering the formula for the magnetoresistance are quantities having such temperature dependence; see e.g. [2]. In view of the presence of  $\tau_{el}$  in the numerator of the formula (7), and the property of proportionality obtained in (15), both the electric and thermal magnetoresistance should increase with a lowering of the temperature  $T$ . Such a behaviour was in fact observed in numerous metals. For example the slopes of dependencies of the thermal magnetoresistance on  $B$  plotted for numerous samples of zinc, lead, cadmium, thallium, silver, tin, and gallium are systematically larger for the lower  $T$  than for higher  $T$ . An exception is represented by indium and a small number of samples of zinc, cadmium, thallium, tin, and gallium for which the slopes of the plots of the thermal magnetoresistance versus  $B$  done for different temperatures merge practically together. In general, the changes of temperatures taken into account for this kind of measurements are relatively quite small [10]. A very good experimental illustration of the rule of the higher thermal magnetoresistance for the lower  $T$  is given in the case of tungsten [11, 12]. This effect is obtained for the transversal kind of the thermal magnetoresistance, but a similar illustration for the temperature dependence of the longitudinal kind is provided in [4].

### 3. Slopes of a Linear Increase of the Thermal Magnetoresistance with $B$

These slopes depend essentially on the relaxation time  $\tau_{el}$  entering tensor (1); see also (7) and (15).  $\tau_{el}$  is a strongly temperature-dependent parameter the behaviour of which can be obtained on the empirical basis; see e.g. [2]. For numerous metals and temperatures

$T < 273$  K it is convenient to apply the ratios

$$\begin{aligned} \frac{1}{r} &= \frac{\rho(T = 273 \text{ K}; B = 0)}{\rho(T_L < 273 \text{ K}; B = 0)} \\ &= \frac{\tau_{\text{el}}(T_L < 273 \text{ K}; B = 0)}{\tau_{\text{el}}(T = 273 \text{ K}; B = 0)} \end{aligned} \quad (17)$$

tabulated in [2]. In general, however, the measurements of  $\Delta k/k(0)$  were done for  $T$  which do not fit  $T_L$  given in [2]. In case of a small difference between the  $T$  and  $T_L$  of [2], the following supplementary correcting factors which couples  $\tau_{\text{el}}$  at a given  $T$  with  $\tau_{\text{el}}$  at  $T = T_L$  can be applied:

$$\begin{aligned} \frac{1}{s} &= \frac{\tau_{\text{el}}(T; B = 0)}{\tau_{\text{el}}(T_L; B = 0)} = \frac{\rho(T_L; B = 0)}{\rho(T; B = 0)} \\ &= \frac{T_L G(\theta/T_L)}{T G(\theta/T)}, \end{aligned} \quad (18)$$

$\theta$  is the Debye temperature of a metal and  $G(\theta/T)$  is the Grueneisen function of the ratio  $\theta/T$ , respectively [19]. The effective  $\tau_{\text{el}}$  for a given  $T$  which replaces  $\tau_{\text{el}}$  for 273 K becomes

$$\tau_{\text{el}}^{\text{eff}}(T) = \frac{\tau_{\text{el}}(T = 273 \text{ K})}{rs}. \quad (19)$$

The experimental  $\tau_{\text{el}}$  for 273 K are taken from [19] and [20]. The relaxation time  $\tau_{\text{el}} = 0.58 \cdot 10^{-14}$  s for the tungsten metal at 273 K is estimated from the empirical data for the electric resistance [21] and  $n_s = 12.7 \cdot 10^{22} \text{ cm}^{-3}$  obtained from the crystallographic data [22].

The slope calculations are facilitated by the relation which couples the cyclotron frequency  $\Omega_0$  and the magnetic field induction  $B$  expressed in kilogauss units. This is

$$\Omega_0 = 1.76 \times 10^{10} B, \quad (20)$$

where the number in (20) is expressed in radians per second; see e.g. [11]. The formula (20) gives the following slope value for a linear dependence of  $\Delta k/k(0)$  on  $B$ :

$$\frac{\Delta k}{Bk(0)} = \frac{1.76 \times 10^{10} \tau_{\text{el}}}{\xi}, \quad (21)$$

on condition  $B$  is expressed in kilogauss units, and  $\tau_{\text{el}}$  is referred to the appropriate temperature. A list of slopes obtained in this way for different metals and different

Table 1. Slope values of a linear increase of the electronic thermal resistance obtained with an increase of  $B$ . Theoretical data of (21) compared with those from the experimental plots [10, 15, 18].  $B$  is expressed in kilo-gauss units. The slope may vary in dependence on that whether a transversal or longitudinal effect of the magnetic field acting on the electric current is taken into account.

Metal	Temperature	Theoretical slope	Experimental slope
Zn	2.5 K	0.56	0.1–0.2
	3.4 K	0.12	0.1
	4.5 K	0.03	0.07
	4.6 K	0.03	0.03–0.2
Cd	2.7 K	3.5	9.5
	3.5 K	1.0	8.9
In	2.2 K	2.8	0.1
	2.8 K	0.88	0.1
	3.25 K	0.4	0.08
W	15 K	0.25	1.0
	20 K	1.0	4.0
Pb	2.7 K	2.6	1.5
	5.3 K	0.1	0.13
	6.4 K	0.035	0.063

temperatures is presented in Table 1. The slopes, calculated with the aid of the quantum-mechanical constant

$$\xi = \frac{1}{2} \quad (22)$$

(see [5]), are compared with the slopes data derived from the experimental plots of the thermal magnetoresistance versus  $B$ . A systematic increase of the slopes accompanied by a decrease of the sample temperature is observed equally on the experimental and theoretical side.

#### 4. Magneto-optical Effect of the Radiation Reflection in a Metal

This kind of effect seems to be not yet discussed by the theory although experimentally the dependence of reflection of radiation incident on metals due to the changes of the magnetic field has been measured a time ago, for example for the antimony metal [23, 24].

In a theoretical approach, an analysis of the relaxation time in a metal is here of importance. In case of a normal incidence of radiation on a planar surface of a metal, the solution of Maxwell's equations provides us with the following formula for a real reflection co-

efficient (see e.g. [25]):

$$R = \frac{(\mathbf{n} - 1)^2 + \mathbf{k}^2}{(\mathbf{n} + 1)^2 + \mathbf{k}^2}. \quad (23)$$

Here  $\mathbf{n}$  and  $\mathbf{k}$  are respectively the real and imaginary parts of the complex refractive index  $\mathbf{N}$ . For metals, the dielectric constant  $\varepsilon$  entering  $\mathbf{N}$  can be neglected, therefore the real and imaginary parts of  $\mathbf{N}$  are then equal in magnitude, giving

$$\mathbf{N} = \mathbf{n} + i\mathbf{k} = \left( \frac{2\pi\sigma}{\omega} \right)^{1/2} (1 + i). \quad (24)$$

Here  $\sigma$  is the conductivity and  $\omega$  the frequency of the incident wave. In the quoted experiments [23, 24], the radiation photon had the energy

$$\hbar\omega = 0.1086 \text{ eV} \quad (25)$$

which gives

$$\omega = 1.65 \cdot 10^{14} \text{ s}^{-1}. \quad (26)$$

In the next step, the carrier concentration in the antimony metal equal to  $n_s = 16.5 \cdot 10^{22} \text{ cm}^{-3}$  [20] provides us with the plasmon frequency

$$\omega_p = \left( \frac{4\pi n_s e^2}{m} \right)^{1/2} \approx 23 \cdot 10^{15} \text{ s}^{-1}. \quad (27)$$

The experiments were performed at temperature of 4 K, so the relaxation time of the electric conductivity is expected to be roughly  $10^2$  times longer than at 273 K (see e.g. [26]). Thus,

$$\tau_{el} = \tau_{el}^{(4 \text{ K})} \cong 10^2 \times 0.55 \times 10^{-14} \text{ s} \approx 10^{-13} \text{ s}, \quad (28)$$

where at 273 K entering (28) is taken from [20].

In effect, we have the relation

$$\frac{1}{\tau_{el}^{(4 \text{ K})}} < \omega < \omega_p \quad (29)$$

which satisfies the relaxation region in which the absorption coefficient falls rapidly [25] but the metal is still strongly reflecting with the coefficient

$$R \cong 1 - \frac{1}{(\pi\sigma\tau_{el})^{1/2}} \approx 1 - \frac{2}{\omega_p\tau_{el}}. \quad (30)$$

The relaxation time  $\tau_{el}$  concerns the electron scattering on the metal defects, so it has no dependence on the

magnetic field. But the scattering on that field is made represented by a separate relaxation time  $\tau_{\text{magn}} = \tau(B)$  which should be combined with  $\tau_{el}$ ; see (4). Moreover, the conduction  $\sigma$  in (30) which depends on  $\tau_{el}$  alone should change its dependence to that on the effective relaxation time  $\tau_{\text{eff}}$ . The details of this point can be examined for the electrons in a metal collected in a single band of states having a nearly free-electron character. If the magnetic field is acting on the electron ensemble in a constant direction, say parallel to the axis  $z$ , the tensor of the electric conductivity becomes [7]

$$\overline{\overline{\sigma}} = \frac{n_s e^2 \tau_{el}}{m(1 + \xi_0^2)} \begin{pmatrix} 1 & \xi_0 & 0 \\ -\xi_0 & 1 & 0 \\ 0 & 0 & 1 + \xi_0^2 \end{pmatrix}. \quad (31)$$

Here  $\tau_{el}$  is the relaxation time characteristic for the electric conductivity in the absence of the magnetic field (see (1)), and  $\xi_0$  is a parameter equal to a product of  $\tau_{el}$  and the cyclotron frequency  $\Omega_0$  in (6); see [7]. Therefore

$$\xi_0 = \tau_{el}\Omega_0. \quad (32)$$

However, when the resistance tensor combined of two tensors (1) and (2) (see (3)) is taken into account, the conductivity tensor (31) is modified into

$$\overline{\overline{\sigma}}^{\text{eff}} = \frac{n_s e^2 \tau_{\text{eff}}}{m(1 + \xi'^2)} \begin{pmatrix} 1 & \xi' & 0 \\ -\xi' & 1 & 0 \\ 0 & 0 & 1 + \xi'^2 \end{pmatrix}, \quad (33)$$

where  $\xi'$  is given by the formula (5). This is so because the resistance tensor obtained in a final step of (3) is the reciprocal tensor of (33); see a similar operation between the resistance and conductivity tensors in case (31) instead of (33) is applied [7]. Moreover, from (5) and (6), we obtain

$$\xi' = \frac{eB}{mc} \tau_{\text{magn}} \frac{\tau_{\text{eff}}}{\tau_{\text{magn}}} = \frac{eB}{mc} \tau_{\text{eff}}. \quad (34)$$

We show below that only an effective relaxation time  $\tau_{\text{eff}}$  can provide us with results for the magneto-effects being in agreement with experiment. For a substitution of the diagonal  $z$ -element from (33), namely

$$\sigma_{zz}^{\text{eff}} = \frac{n_s e^2 \tau_{\text{eff}}}{m}, \quad (35)$$

into (30), and the respective change of  $\tau_{el}$  into  $\tau_{eff}$ , gives instead of (30) the formula

$$R = R(B) \cong 1 - \frac{2}{\omega_p \tau_{eff}} \quad (36)$$

$$= 1 - \frac{2}{\omega_p \tau_{el}} - \frac{2}{\omega_p \tau_{magn}},$$

where  $\tau_{magn}$  depends on  $B$  because of (6) and (8). In experiments [23, 24] the ratio

$$\frac{B(B) - R(0)}{R(0)} = \frac{\frac{2}{\omega_p \tau_{magn}}}{1 - \frac{2}{\omega_p \tau_{el}}} \quad (37)$$

is measured. The result in (37) is attained because of the value  $R(0) = R(B=0)$  presented in the formula (30).

With the data taken from (26) and (28), the reflection coefficient

$$R(0) \cong 1 - \frac{2}{23 \times 10^{15} \times 10^{-13}} \cong 1 - \frac{1}{10^3} \quad (38)$$

is a number close to unity. On the other hand, the numerator in (37) is a number proportional to  $B$ ; see (8). This is in fact a measured result in [23, 24] obtained till very high  $B$  on condition the absolute value of  $R(B) - R(0)$  is taken into account. However, a result for the proportionality coefficient  $\alpha$  multiplying  $B$  in the formula

$$\frac{|R(B) - R(0)|}{R(0)} = \alpha B \quad (39)$$

is rather poor: experimentally the slope  $\alpha$  in (39) attains the value of about 0.2 for  $B = 110$  kG, but the theoretical slope is much smaller, for it is only about 0.0004. To some extent, corrections due to the effective electron mass and the concentration  $n_s$  may be expected to improve this situation.

A similar linear dependence of the reflectivity data on  $B$  has been observed also for graphite [27, 28], both with the aid of the conventional spectrometer and with laser. Here, however, a breakdown of reflectivity at very large  $B$ , absent in the antimony metal, is taking place.

Beyond of the main dependence on  $B$ , also small oscillations in the magnetoreflexion are obtained [23, 24]. They can be identified with the optical de Haas–Schubnikow effect which occurs when the Landau levels are crossed by the Fermi surface because of the

change of the magnetic field. A characteristic point is here that the periods of the oscillations obtained separately from the magnetoreflexion and magnetoresistance are much similar. This can be expected from the fact that both these effects are dependent on the induction  $B$  in the same way: the magnetoreflexion in (37) and (39) is proportional to  $\tau_{magn}^{-1}$ , but the same dependence on  $\tau_{magn}$  holds also for the magnetoresistance; see formula (39) in [5].

It should be noted that in order to obtain a positive change (increase) of reflectance  $R$  with increase of  $B$ , a negative sign has been applied before  $B$  in the definition of  $\Omega_0$  given in (6). Such a definition is in fact introduced for  $\Omega_0$  in some textbooks; see e.g. [7]. But a quantum-mechanical derivation of the parameter value  $\xi$  given in [5] is insensitive to the choice of the sign for  $B$ .

## 5. Extrema in the Energy Loss Spectrum Connected with the Cyclotron Resonance

The conductivity tensor (31), and its modification (33), can be the basis of calculations also in case when the electric field is modulated by the frequency  $\omega$ :

$$\vec{E} = \vec{E}_0 e^{i\omega t}; \quad (40)$$

see [29]. The power  $P$  absorbed by the electrons from the electromagnetic field referred to the total power of the linear wave  $P_0$  is [29]

$$\frac{P}{P_0} = \frac{1 + (\Omega_0^2 + \omega^2)\tau^2}{[1 + (\Omega_0^2 - \omega^2)\tau^2]^2 + 4\omega^2\tau^2} \quad (41)$$

$$= \frac{1 + x + q}{(1 + x - q)^2 + 4q},$$

where  $x = \Omega_0^2\tau^2$  and  $q = \omega^2\tau^2$ .  $P_0$  corresponds to the situation when  $B = \Omega_0 = 0$  and  $\omega = 0$ , leading to the ratio of (41) equal to unity [29].

The extremum of (41) is obtained from the condition

$$\frac{\partial}{\partial x} \left( \frac{P}{P_0} \right) = \frac{(1 + x - q)^2 + 4q - 2(1 + x - q)(1 + x + q)}{[(1 + x - q)^2 + 4q]^2} = 0 \quad (42)$$

which gives

$$x_{1,2} = -1 - q \pm 2\sqrt{q + q^2}. \quad (43)$$

A physical background of  $x$  implies  $x > 0$ , so  $q \gg 1$ .



Metal	Exper. $\omega$ (in Gcps)	Exper. $B$ at resonance (in kG)	Exper. $\Omega_0^{\text{ext}}$ (in rad/s)	Theor. $\Omega_0^{\text{ext}}$ (in rad/s)	Theor. $\frac{m^{\text{eff}}}{m}$ from (46)	Exper. $\frac{m^{\text{eff}}}{m}$
				see (44a)		
Al [31]	35.5	10	$1.76 \times 10^{11}$	$3.15 \times 10^{11}$	0.56	0.11–0.24
Zn [32]	72	13	$2.6 \times 10^{11}$	$6.4 \times 10^{11}$	0.4	1.3
Cd [33]	72	15	$2.6 \times 10^{11}$	$6.4 \times 10^{11}$	0.4	0.4–0.76
Sb [34]	23.5	1	$0.18 \times 10^{11}$	$2.1 \times 10^{11}$	0.09	0.05–1.03

Table 2. Observed cyclotron frequency of the extremal absorption for a signal having the frequency  $\omega$  compared with the frequency calculated from the formula (44a). The effective electron mass calculated from the ratio of both frequencies is compared with the effective mass obtained from experiments given in the references quoted in the last column of the Table.

In effect, the solution in (43) corresponding to the extremal value of  $P/P_0$  becomes approximately

$$x^{\text{ext}} = (\Omega_0^{\text{ext}})^2 \tau^2 \cong 2q = 2\omega^2 \tau^2 \quad (44)$$

or

$$\Omega_0^{\text{ext}} \approx \sqrt{2} \omega. \quad (44a)$$

The result in (44a) can be compared with experiment. In fact, the measured positions of the extrema in the energy loss spectrum do not differ much from the modulation frequency  $\omega$ . Assuming that the effective electron mass is entering the frequency  $\Omega_0^{\text{ext}}$ , this frequency is calculated according to the equation

$$\Omega_0^{\text{ext}(\text{theor})} = \frac{eB}{m^{\text{eff}}_c} = \sqrt{2} \omega. \quad (45)$$

It should be noted that, contrary to former theoretical approaches [30], no empirical parameters, especially those referring to the relaxation time, are used in the present calculations.

The formula (45) can serve in an estimate of the electron effective mass from the relation

$$\frac{m^{\text{eff}}}{m} = \frac{\Omega_0^{\text{ext}(\text{observed})}}{\Omega_0^{\text{ext}(\text{theor})}}, \quad (46)$$

on condition an ordinary electron mass is applied in expressing the observed resonance frequency with the aid of the strength of the magnetic field.

In Table 2, we compare the calculated data of the resonance frequency  $\Omega_0^{\text{ext}(\text{theor})}$  obtained from (45), the imposed modulation frequency  $\omega$ , and the experimental resonance frequency  $\Omega_0^{\text{ext}(\text{observed})}$  (in radians per second). By taking into account (46), the effective electron masses for aluminium, zinc, antimon, and cadmium metals can be deduced.

A better agreement between the theoretical and experimentally estimated effective mass seems to be obtained for cadmium and antimon, a worse result is for

aluminium and zinc. It should be noted, however, that no geometrical parameters concerning the metal sample are taken into account in deriving the formula (45).

## 6. Summary

In the first step, the thermal electron magnetoresistance of metals has been theoretically examined. This effect is approximately proportional to the electric magnetoresistance, therefore its dependence on the magnetic induction is linear, in agreement with the observed data.

A more delicate problem is a dependence of the slope of increase of the thermal magnetoresistance on temperature. Roughly that behaviour should be also similar to the electric magnetoresistance which means a proportionality of the thermal effect to the relaxation time  $\tau_{el}$  of the electric resistance. Since  $\tau_{el}$  increases with a lowering of temperature, a similar property should apply for the thermal magnetoresistance. This lowering effect of the temperature increase on the electric magnetoresistance and thermal magnetoresistance is in fact confirmed by the data obtained in experiments.

The next point considered was the change of the metal reflectance due to the change of the strength of the magnetic induction  $B$ . In this case the part  $\tau_{\text{magn}}$  of the effective relaxation time  $\tau_{\text{eff}}$  dependent on  $B$  plays a dominant role. Because the reflectance occurs to be inversely proportional to  $\tau_{\text{magn}}$ , and  $\tau_{\text{magn}}^{-1} \sim B$ , the proportionality of the reflectance to  $B$  – confirmed by experiment – is an immediate result of the theory.

The last item considered is the energy loss spectrum of the electron ensemble perturbed by a periodic signal. When examined in the presence of an external magnetic field, the position of the extremum of the loss spectrum with respect to the strength of the field is found to become independent of any empirical parameter of the electron medium, for example the relaxation

time. This position is defined solely by the signal frequency multiplied by a constant number. An identification of the frequency calculated at the extremum position with the frequency supplied by the cyclotron reso-

nance allowed us to deduce the effective electron mass in several metal cases. These numbers are not much different from those estimated directly on the basis of other experimental data.

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