# Influence of the Dynamic Shielding on the Collisional Entanglement Fidelity in Strongly Coupled Semiclassical Plasmas

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The influence of the dynamic plasma shielding on the collisional entanglement fidelity is investigated in strongly coupled semiclassical plasmas. The partial wave analysis with the effective dynamic screening length is employed to obtain the dynamic entanglement fidelity as a function of collision energy, de Broglie wavelength, Debye length, and thermal energy. The results show that the collisional entanglement fidelity increases with increasing plasma temperature as well as de Broglie wavelength and, however, decreases with an increase of the Debye length. It is also found that the dynamic screening effect suppresses the collisional entanglement fidelity in strongly coupled semiclassical plasmas. In addition, it is found that the entanglement fidelity decreases with increasing de Broglie wavelength and, however, increases with increasing thermal energy. It is also found that the thermal effect on the entanglement fidelity would be more significant in the domain of low-collision energies.

*Key words:* Dynamic Shielding; Collisional Entanglement Fidelity; Semiclassical Plasmas.

The collision process in plasmas [1 – 5] has been of a great interest since the process is known as the one of the major atomic processes and also has provided useful information on the electrical conductivity and mobility in plasmas. It is also shown that the entanglement fidelity in the collision process has received a considerable attention since it has been shown that the quantum correlation phenomenon [6] plays an important role for understanding the quantum measurements and information processing in various quantum systems. Recently, the physical properties of strongly coupled plasmas have been extensively investigated since the interiors of astrophysical compact objects and inertial confinement fusion plasmas would be classified as strongly coupled plasmas [7 – 11]. In these strongly coupled semiclassical plasmas, the interaction potential would not be characterized by the standard Debye–Hückel model obtained by the linearization of the Poisson equation with the Boltzmann distribution owing to the multi-particle correlations caused by the collective screening and quantum-mechanical effects. In addition, it has been shown that the statically shielding model for particle collisions would not be reliable to describe the screened interaction potential in plasmas when the collision velocity is comparable to or smaller than the velocity of the plasma electron since the projectile particle would polarize the surrounding plasma particles. Hence, the dynamic motion of plasma particles should be then taken into account in these situations in order to properly represent the plasma screening effects on the various collision processes in plasmas. Hence, it would be expected that the dynamically screened collisional entanglement fidelities would be quite different from the statically screened entanglement fidelities for the elastic electron–ion collisions in strongly coupled semiclassical plasmas. Thus, in this paper, we investigate the influence of the dynamic plasma shielding on the entanglement fidelity for the elastic electron–ion collision in strongly coupled semiclassical plasmas. The partial wave analysis [12] with the effective dynamic screening length [13] is employed to explore the screened collisional entanglement fidelity in strongly coupled semiclassical plasmas as a function of collision energy, de Broglie wavelength, Debye length, and thermal energy.

In quantum scattering processes, the stationary-state Schrödinger equation [14] for the reduced interaction potential $U(\mathbf{r})$ is written as

$$(\nabla^2 + k^2)\Psi(\mathbf{r}) = U(\mathbf{r})\Psi(\mathbf{r})\,, \tag{1}$$

where $\Psi(\mathbf{r})$ represents the scattered wave function, $k[= (2\mu E/\hbar^2)^{1/2}]$ is the wave number, $\mu$ the reduced mass of the collision system, $E(= \mu v^2/2)$ the collision energy, $v$ the collision velocity, $\hbar$ the rationalized Planck constant, and $V(\mathbf{r})[\equiv (\hbar^2/2\mu)U(\mathbf{r})]$ the interaction potential. Using the method of partial waves [14], the scattered wave function $\Psi(\mathbf{r})$ is represented as the decomposition into modes of a finite angular momentum quantum number $l$:

$$\Psi(\mathbf{r}) = \sum_{l=0}^{\infty} D_l(k)(2l+1)\mathrm{i}^l P_l(\cos\theta)u_l(r)\,, \tag{2}$$

where $D_l(k)$ is the expansion coefficient, $P_l(\cos\theta)$ the Legendre polynomial, and $u_l(r)$ the solution of the radial part of the Schrödinger equation:

$$\left[\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}}{\mathrm{d}r}\right) - \frac{l(l+1)}{r^2} - U(r) + k^2\right]u_l(r) = 0\,. \tag{3}$$

Here, the asymptotic form of the radial wave function $u_l(r)$ can be represented by the scattering phase shift $\delta_l$ such as $u_l(r) \propto \sin(kr - \pi l/2 + \delta_l)/kr$. For a central potential field $U(r)$, the radial wave function $u_l(r)$ and expansion coefficient $D_l(k)$ are, respectively, obtained by the following forms [14]:

$$\begin{aligned} u_l(r) = j_l(kr) + \bigg[&kn_l(kr)\int_0^r \mathrm{d}r' r'^2 U(r') j_l(kr')\, u_l(r') \\ &+ kj_l(kr)\int_r^{\infty} \mathrm{d}r' r'^2 U(r') n_l(kr') u_l(r')\bigg]\,, \end{aligned} \tag{4}$$

$$\begin{aligned} D_l(k) = &(2\pi)^{-3/2} \\ &\cdot\left[1 + \mathrm{i}k\int_0^{\infty} \mathrm{d}r r^2 U(r) j_l(kr) u_l(r)\right]^{-1}\,, \end{aligned} \tag{5}$$

where $j_l(kr)$ and $n_l(kr)$ are the spherical Bessel and Neumann functions of order $l$, respectively. It has been shown that the entanglement fidelity $f(k)$ for collision processes can be represnetd by the absolute square of the scattered wave function such as $f(k) \propto \left|\int \mathrm{d}^3\mathbf{r}\Psi(\mathbf{r})\right|^2$ by a recent excellent work by Mishima, Hayashi, and Lin [15]. Since the $s$-wave state provides the main contribution to the low-energy projectiles, the entanglement fidelity for the low-energy collisions is obtained by

$$\begin{aligned} f(k) \propto &\left|\int_0^{\infty} \mathrm{d}r r^2 j_0(kr)\right|^2 \\ &\cdot\left[1 + \left|k\int_0^{\infty} \mathrm{d}r r^2 U(r) j_0(kr)\right|^2\right]^{-1}\,. \end{aligned} \tag{6}$$

Recently, the dynamic screening length [13] in non-ideal plasmas has been obtained as a function of the velocity such as $r_0(v)[= (1 + v^2/v_{\mathrm{T}}^2)^{1/2} r_{\mathrm{D}}]$, where $v$ is the collision velocity, $v_{\mathrm{T}}(= \sqrt{k_{\mathrm{B}}T/m})$ the electron thermal velocity, $k_{\mathrm{B}}$ the Boltzmann constant, $T$ the plasma temperature, $v$ the velocity of the projectile electron, and $r_{\mathrm{D}}$ the standard Debye length. This velocity dependence of the plasma screening length $r_0(v)$ is readily understood since the dynamic plasma screening effect turns out to be the static plasma screening case such as $r_0(v) \to r_{\mathrm{D}}$ when the velocity of the projectile electron is smaller than the electron thermal velocity. In addition, an accurate form of the screened effective pseudopotential [4] of the particle interactions in strongly coupled semiclassical plasmas taking into account both the quantum-mechanical and plasma screening effects was obtained on the basis of the dielectric response function analysis. Hence, the velocity dependent dynamic interaction potential $V_{\mathrm{Dy}}(r,v)$ between the projectile electron and target ion with nuclear charge $Ze$ in strongly coupled semiclassical plasmas based on the Ramazanov–Dzhumagulova model [4] with the dynamic screening length $r_0(v)$ is obtained by

$$\begin{aligned} V_{\mathrm{Dy}}(r,v) = &-\frac{Ze^2}{[1 - 4\lambda^2/r_0^2(v)]^{1/2}} \\ &\cdot\left\{\frac{\exp[-A(\lambda, r_0(v))r]}{r} - \frac{\exp[-B(\lambda, r_0(v))r]}{r}\right\}\,, \end{aligned} \tag{7}$$

where $\lambda(= \hbar/\sqrt{2\pi m k_{\mathrm{B}}T})$ is the thermal de Broglie wavelength of the electron; the velocity dependent screening parameters are given by $A(\lambda, r_0(v)) \equiv [1 - \sqrt{1 - 4\lambda^2/r_0^2(v)}/(2\lambda^2)]^{1/2}$ and $B(\lambda, r_0(v)) \equiv [1 + \sqrt{1 - 4\lambda^2/r_0^2(v)}/(2\lambda^2)]^{1/2}$, respectively. This velocity dependent dynamic interaction potential $V_{\mathrm{Dy}}(r,v)$ is valid for $2\lambda < (1 + v^2/v_{\mathrm{T}}^2)^{1/2} r_{\mathrm{D}}$. If we neglect the quantum-mechanical effect in plasmas, the effective dynamic interaction potential (7) turns out to be the classical dynamic Debye–Hückel form: $V_{\mathrm{Dy}}(r,v) \to -(Ze^2/r)\,\mathrm{e}^{-r/r_0(v)}$. Hence, the dynamic plasma screening effects on the collisional entanglement fidelity for the elastic collisions in strongly coupled semiclassical plasmas is investigated by the dynamic fidelity ratio $R_{\mathrm{F}}[\equiv f_{\mathrm{Dy}}(v,v_{\mathrm{T}},r_{\mathrm{D}},\lambda)/f_{\mathrm{C}}(v)]$ determined by the ratio of the dynamic collisional entanglement fidelity $f_{\mathrm{Dy}}(v,v_{\mathrm{T}},r_{\mathrm{D}},\lambda)$ using the effective dynamic interaction potential $V_{\mathrm{Dy}}(r,v)$ (7) in strongly coupled semiclassical plasmas to the Coulomb entanglement fidelity $f_{\mathrm{C}}(v)$ using the pure Coulomb interaction ($V_{\mathrm{C}} = -Ze^2/r$):

$$\begin{aligned} R_{\mathrm{F}}(\bar{E},\bar{E}_{\mathrm{T}},\bar{r}_{\mathrm{D}},\bar{\lambda}) &= \left[\left|-\frac{2Ze^2\mu}{\hbar^2}\int_0^\infty \mathrm{d}r\sin(kr)\right|^2+1\right] \\ &\Bigg/ \left\{\left|-\frac{2Ze^2\mu}{[1-4\lambda^2/r_0^2(v)]^{1/2}}\int_0^\infty \mathrm{d}r\sin(kr)\right.\right. \\ &\cdot\left[\exp\left[-\left(1-\sqrt{1-4\lambda^2/r_0^2(v)}\right)^{1/2} r/(\sqrt{2}\lambda)\right]\right. \\ &\left.\left.\left.\cdot\exp\left[-\left(1+\sqrt{1-4\lambda^2/r_0^2(v)}\right)^{1/2} r/(\sqrt{2}\lambda)\right]\right]\right|^2+1\right\} \\ &= \frac{\bar{E}+4}{\bar{E}+\left[\frac{2\bar{E}(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2}{\bar{E}^2(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2\bar{\lambda}^2+\bar{E}(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2+1}\right]^2}, \end{aligned} \tag{8}$$

where $\bar{E}(\equiv E/Z^2\mathrm{Ry})$ is the scaled projectile energy, $\mathrm{Ry}(= m\mathrm{e}^4/2\hbar^2 \cong 13.6\ \mathrm{eV})$ the Rydberg constant, $m$ the mass of the electron, $\bar{E}_{\mathrm{T}}(\equiv k_{\mathrm{B}}T/2Z^2\mathrm{Ry})$ the scaled thermal energy, $\bar{\lambda}(\equiv \lambda/a_Z)$ the scaled thermal de Broglie wavelength, $a_Z(= a_0/Z)$ the first Bohr radius of the hydrogenic ion with nuclear charge $Ze$ and Bohr radius $a_0(= \hbar^2/me^2)$, and $\bar{r}_{\mathrm{D}}(\equiv r_{\mathrm{D}}/a_Z)$ is the scaled Debye length. The dynamic plasma screening effects on the collisional fidelity ration are then investigated by the following form of the dynamic screening function $F_{\mathrm{R}}(\bar{E},\bar{E}_{\mathrm{T}},\bar{r}_{\mathrm{D}},\bar{\lambda})$ defined as

$$F_{\mathrm{R}}(\bar{E},\bar{E}_{\mathrm{T}},\bar{r}_{\mathrm{D}},\bar{\lambda}) = \frac{\left[\frac{2\bar{E}(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2}{\bar{E}^2(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2\bar{\lambda}^2+\bar{E}(1+\bar{E}/\bar{E}_{\mathrm{T}})\bar{r}_{\mathrm{D}}^2+1}\right]^2+\bar{E}}{\left(\frac{2\bar{E}\bar{r}_{\mathrm{D}}^2}{\bar{E}^2\bar{r}_{\mathrm{D}}^2\bar{\lambda}^2+\bar{E}\bar{r}_{\mathrm{D}}^2+1}\right)^2+\bar{E}}. \tag{9}$$

This dynamic screening function $F_{\mathrm{R}}(\bar{E},\bar{E}_{\mathrm{T}},\bar{r}_{\mathrm{D}},\bar{\lambda})$ of the collisional entanglement fidelity can be regarded as the quantitative measure of change in the emission and scattering spectra owing to the influence of the dynamic plasma screening in strongly coupled semiclassical plasmas. Recently, the influence of the static quantum screening [16] on the entanglement fidelity for the elastic collision was explored in electron-ion quantum plasmas. However, the dynamic plasma screening effect on the collisional entanglement fidelity in strongly coupled semiclassical plasmas has not been investigated as yet. Hence, it is expected that the dynamic screening function prides us the useful information on the collision and radiation processes in semiclassical plasmas. In strongly coupled semiclassical plasmas [17] consisting of electrons and ions, the ranges of the number density $n$ and the temperature $T$ are known to be about $10^{20}-10^{24}\ \mathrm{cm}^{-3}$ and $5\cdot 10^4 - 10^6$ K, respectively.

Figure 1 shows the dynamic collisional entanglement fidelity ratio $R_{\mathrm{F}}(\bar{E},\bar{E}_{\mathrm{T}},\bar{r}_{\mathrm{D}},\bar{\lambda})$ in strongly coupled semiclassical plasmas as a function of the scaled thermal de Broglie wavelength $\bar{\lambda}$ for various values of the thermal energy $\bar{E}_{\mathrm{T}}$. As it is seen, the entanglement fidelity ratio increases with increasing de Broglie wavelength. Hence, we have understood that the quantum-mechanical effect enhances the entanglement fidelity in strongly coupled semiclassical plasmas. It is also found that the collisional entanglement fidelity decreases with increasing de Broglie wavelength and, however, increases with increasing thermal energy. Figure 2 represents the entanglement fi-

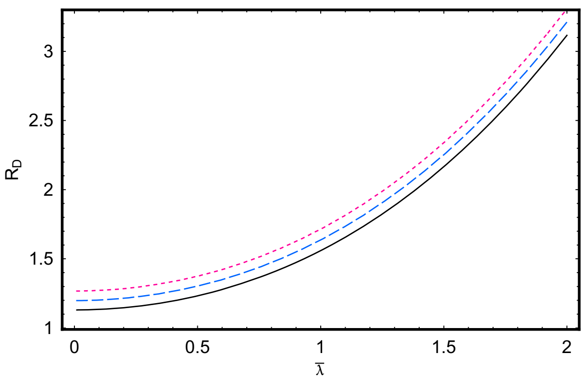


Fig. 1 (colour online). Collisional entanglement fidelity ratio $R_{\mathrm{F}}$ in strongly coupled semiclassical plasmas as a function of the scaled thermal de Broglie wavelength $\bar{\lambda}$ when $\bar{E} = 0.2$ and $\bar{r}_{\mathrm{D}} = 5$. Solid line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.1$; dashed line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.2$; dotted line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.4$.

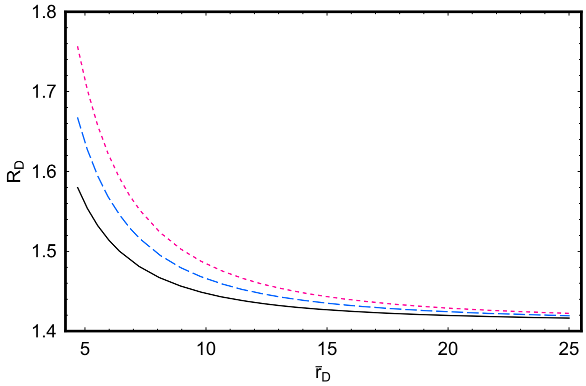


Fig. 2 (colour online). Collisional entanglement fidelity ratio $R_{\mathrm{F}}$ as a function of the scaled Debye length $\bar{r}_{\mathrm{D}}$ when $\bar{E} = 0.2$ and $\bar{\lambda} = 1$. Solid line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.1$; dashed line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.2$; dotted line: fidelity ratio for $\bar{E}_{\mathrm{T}} = 0.4$.

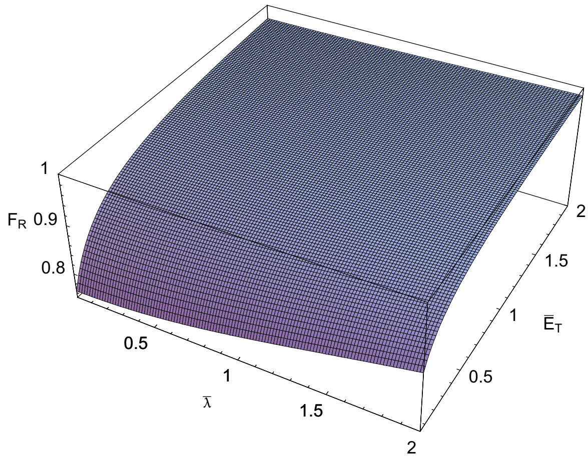


Fig. 3 (colour online). Surface plot of the dynamic screening effect $F_{\mathrm{R}}$ on the collisional entanglement fidelity as a function of the scaled thermal de Broglie wavelength $\bar{\lambda}$ and scaled thermal energy $\bar{E}_{\mathrm{T}}$ when $\bar{E} = 0.2$ and $\bar{r}_{\mathrm{D}} = 5$.

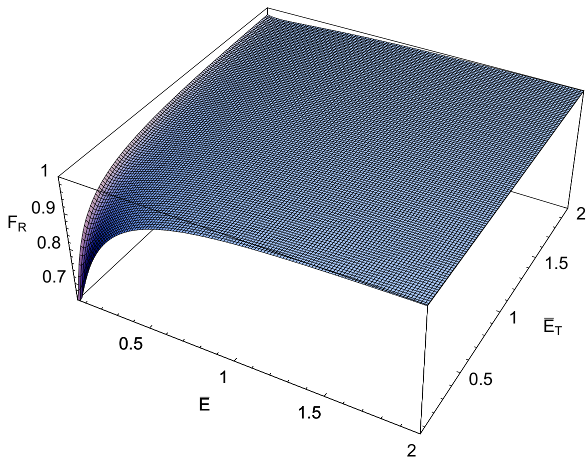


Fig. 4 (colour online). Surface plot of the dynamic screening effect $F_{\mathrm{R}}$ on the collisional entanglement fidelity as a function of the scaled collision energy $\bar{E}$ and scaled thermal energy $\bar{E}_{\mathrm{T}}$ when $\bar{r}_{\mathrm{D}} = 5$ and $\bar{\lambda} = 1$.

delity ratio $R_{\mathrm{F}}(\bar{E}, \bar{E}_{\mathrm{T}}, \bar{r}_{\mathrm{D}}, \bar{\lambda})$ as a function of the scaled Debye length $\bar{r}_{\mathrm{D}}$ for various values of the thermal energy $\bar{E}_{\mathrm{T}}$. As shown in this figure, the entanglement fidelity ratio decreases with an increase of the Debye length. Hence, it is expected that the plasma shielding effect enhances the collisional entanglement fidelity in plasmas. It is also found that the thermal effect on the entanglement fidelity decreases with increasing Debye length. Figure 3 represents the dynamic screening effect $F_{\mathrm{R}}(\bar{E}, \bar{E}_{\mathrm{T}}, \bar{r}_{\mathrm{D}}, \bar{\lambda})$ on the collisional entanglement fidelity as a function of the scaled thermal de Broglie wavelength $\bar{\lambda}$ and scaled thermal energy $\bar{E}_{\mathrm{T}}$. As it is seen, the dynamic plasma screening effect suppresses the collisional entanglement fidelity in strongly coupled semiclassical plasmas. From this figure, we have found that the dynamic plasma screening effect on the collisional entanglement fidelity decreases with an increase of the de Broglie wavelength, i.e., increase of the quantum-mechanical effect. Hence, it can be expected that the dynamic plasma screening effects on the collisional entanglement fidelity in classical plasmas are stronger than those in semiclassical plasmas. It is also found that the dynamic plasma screening effect decreases with increasing thermal energy. It is then also expected that the dynamic plasma screening effects on the entanglement fidelity in warm plasmas are stronger than those in hot plasmas. Figure 4 represents the dynamic screening effect $F_{\mathrm{R}}(\bar{E}, \bar{E}_{\mathrm{T}}, \bar{r}_{\mathrm{D}}, \bar{\lambda})$ on the collisional entanglement fidelity as a function of the scaled collision energy $\bar{E}$ and scaled thermal energy $\bar{E}_{\mathrm{T}}$. As shown, the thermal effect on the entanglement fidelity are more significant in the domain of low-collision energies. In addition, the energy dependence on the entanglement fidelity decreases with an increase of the thermal energy. From these results, we have found that the dynamic plasma screening effect plays a significant role on the collisional entanglement fidelity in strongly coupled semiclassical plasmas. It is also found that the use of the accurate dynamic screening interaction potential is essential for investigating various collision and radiation processes in dense plasmas. These results provide useful information on the relocation and transfer of the quantum information for collision processes in strongly coupled semiclassical plasmas.

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