Radiative and Porous Effects on Free Convection Flow near a Vertical Plate that Applies Shear Stress to the Fluid

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Z. Naturforsch. **68a,** 130 – 138 (2013) / DOI: 10.5560/ZNA.2012-0083
Received August 27, 2012 / published online February 15, 2013

General solutions for the unsteady free convection flow of an incompressible viscous fluid due to an infinite vertical plate that applies a shear stress $f(t)$ to the fluid are established when thermal radiation and porous effects are taken into consideration. They satisfy all imposed initial and boundary conditions and can generate a large class of exact solutions corresponding to different motions with technical relevance. The velocity is presented as a sum of thermal and mechanical components. Finally, some special cases are brought to light, and effects of pertinent parameters on the fluid motion are graphically underlined.

Key words: Porous Medium; Free Convection; Thermal Radiation; Shear Stress; General Solutions.

1. Introduction

Radiative convective flows of an incompressible viscous fluid past a vertical plate have applications in many industrial processes. Radiative heat transfer plays an important role in manufacturing industries, filtration processes, drying of porous materials in the textile industry, solar energy collectors, satellites and space vehicles, etc. Unsteady convective radiative flows have important applications in geophysics, geothermics, chemical and ceramics processing. Many studies analyzing effects of thermal radiation in convection flows through porous media have recently appeared. A short presentation of the main results till 2007 is given by Ghosh and Beg [1] who studied the convective radiative heat transfer past a hot vertical surface in porous media.

In the last years, problems of free convection and heat transfer through porous media have attracted the attention of many researchers. Flows through porous media have numerous engineering and geophysical applications in chemical engineering for filtration and purification processes, agriculture engineering to study the underground water resources, petroleum technology, and so on. Among the most recent and interesting results on this line we remember here the work by Chaudary et al. [2], Toki [3], Rajesh [4], Narahari [5], Chandrakala and Bhaskar [6], Narahari and Ishak [7], Samiulhaq et al. [8] and references therein. However, it is worth pointing out that all these papers have a common specific feature. Namely, they solve problems in which the velocity is given on the boundary.

Generally, there are three types of boundary value problems in fluid mechanics: i) velocity is given on the boundary; ii) shear stress is given on the boundary; iii) mixed boundary value problems. From theoretical and practical point of view, all three types of boundary conditions are identically important; as in some problems what is specified is the force applied on the boundary. It is also important to bear in mind that the ‘no slip’ boundary condition may not be necessarily applicable to flows of polymeric fluids that can slip or slide on the boundary. Thus, the shear stress boundary condition is particularly meaningful. To the best of our knowledge, the first exact solutions for motions of non-Newtonian fluids in which the shear stress is given on the boundary are those of Waters and King [9] and Bandelli et al. [10]. In the last time many similar solutions have been established by different authors [11 – 14]. However, in all these papers, the radiative and porous effects have not been taken into consideration.

The purpose of this work is to provide exact solutions for the unsteady free convection flow of an incompressible viscous fluid over an infinite vertical plate that applies a time-dependent shear stress $f(t)$ to the fluid. The viscous dissipation is neglected but radiative and porous effects are taken into consideration. General solutions that have been obtained satisfy all imposed initial and boundary conditions and are not common in the literature. They generate a large class of exact solutions for different motion problems that are similar to fluid motions in which velocity is given on the boundary. To illustrate their theoretical and practical importance, three special cases are considered, and the effects of pertinent parameters on the dimensionless velocity and temperature are graphically underlined.

2. Mathematical Formulation

Let us consider the unsteady flow of an incompressible viscous radiating fluid over an infinite hot vertical plate embedded in a porous medium. The x-axis of the Cartesian coordinate system is taken along the plate in the vertical direction and the y-axis is normal to the plate. Initially the plate and the fluid are at the same temperature T_∞ in a stationary condition. After time $t = 0^+$, the plate applies a time-dependent shear stress $f(t)$ to the fluid along the x-axis. At the same time, the temperature of the plate is raised to T_W. The radiative heat flux is considered to be negligible in the x-direction in comparison to the y-direction. The fluid is grey absorbing–emitting radiation but no scattering medium. Assuming that the viscous dissipation is negligible and using the usual Boussinesq's approximation, the unsteady flow is governed by the following equations [1]:

$$\frac{\partial u(y,t)}{\partial t} = \nu \frac{\partial^2 u(y,t)}{\partial y^2} + g\beta\,[T(y,t) - T_\infty] - \frac{\nu}{K} u(y,t); \quad y,t > 0, \tag{1}$$

$$\rho C_\mathrm{p} \frac{\partial T(y,t)}{\partial t} = k \frac{\partial^2 T(y,t)}{\partial y^2} - \frac{\partial q_\mathrm{r}(y,t)}{\partial y}; \quad y,t > 0, \tag{2}$$

where u, T, ν, g, β, K, ρ, C_p, k, and q_r are the velocity of the fluid, its temperature, the kinematic viscosity of the fluid, the gravitational acceleration, the coefficient of thermal expansion, the permeability of the porous medium, the constant density of the fluid, the specific heat at constant pressure, the thermal conductivity of the fluid, and the radiative heat flux, respectively.

Assuming that no slip appears between the plate and fluid, the appropriate initial and boundary conditions are

$$\begin{aligned} &u(y,0) = 0, \;\; T(y,0) = T_\infty \;\; \text{for} \;\; y \geq 0, \\ &\left.\frac{\partial u(y,t)}{\partial y}\right|_{y=0} = \frac{f(t)}{\mu}, \;\; T(0,t) = T_\mathrm{W} \;\; \text{for} \;\; t > 0, \\ &u(y,t) \to 0, \;\; T(y,t) \to T_\infty \;\; \text{as} \;\; y \to \infty, \end{aligned} \tag{3}$$

where $\mu = \rho\nu$ is the coefficient of viscosity, and the function $f(t)$ satisfies the condition $f(0) = 0$.

In the following, we adopt the Rosseland approximation for the radiative flux q_r [1, 7, 8, 15–17], namely

$$q_\mathrm{r} = -\frac{4\sigma}{3k_\mathrm{R}} \frac{\partial T^4}{\partial y}, \tag{4}$$

where σ is the Stefan–Boltzmann constant, and k_R is the mean spectral absorption coefficient or the Rosseland mean attenuation coefficient [18]. Assuming that the temperature difference between the fluid temperature T and the free stream temperature T_∞ is sufficiently small, expanding T^4 in a Taylor series about T_∞, and neglecting higher order terms, we find that

$$T^4 \approx 4T_\infty^3 T - 3T_\infty^4. \tag{5}$$

It is worth pointing out that (5) is widely used in computational fluid dynamics involving absorption problems [19]. Introducing (5) into (4) and using the result in the governing equation (2), we find that

$$\mathrm{Pr} \frac{\partial T(y,t)}{\partial t} = \nu(1 + N_\mathrm{r}) \frac{\partial^2 T(y,t)}{\partial y^2}; \quad y,t > 0, \tag{6}$$

where the Prandtl number Pr and the radiation–conduction parameter N_r are defined by [1, 20]

$$\mathrm{Pr} = \frac{\mu C_\mathrm{p}}{k}, \;\; \text{respectively} \;\; N_\mathrm{r} = \frac{16\sigma T_\infty^3}{3kk_\mathrm{R}}. \tag{7}$$

In the following, the dimensionless solutions of coupled partial differential equations (1) and (6) with the initial and boundary conditions (3) will be determined by means of Laplace transforms.

3. Dimensionless Analytical Solutions

In order to obtain non-dimensional forms of governing equations (1) and (6), and to reduce the number of essential parameters, let us introduce the following dimensionless quantities

$$u^*=\frac{u}{U},\ T^*=\frac{T-T_\infty}{T_W-T_\infty},\ y^*=\frac{U}{\nu}y,\ t^*=\frac{U^2}{\nu}t,$$
$$f^*(t^*)=\frac{1}{\rho U^2}f\left(\frac{\nu}{U^2}t^*\right),\ \text{and}\ K_p=\frac{\nu^2}{U^2}\frac{1}{K},\tag{8}$$

where K_p is the inverse permeability parameter for the porous medium. In order to reduce the number of essential parameters, let us chose the reference velocity $U=[g\beta\nu(T_W-T_\infty)]^{1/3}$. Introducing (8) into (1) and (6) and dropping out the star notation, we find the non-dimensional governing equations in the suitable forms

$$\frac{\partial u(y,t)}{\partial t}=\frac{\partial^2 u(y,t)}{\partial y^2}+T(y,t)-K_p u(y,t);\ y,t>0,\tag{9}$$
$$\mathrm{Pr_{eff}}\frac{\partial T(y,t)}{\partial t}=\frac{\partial^2 T(y,t)}{\partial y^2};\ y,t>0,\tag{10}$$

where $\mathrm{Pr_{eff}}=\mathrm{Pr}/(1+N_r)$ is the effective Prandtl number [20, (10)].

The corresponding boundary conditions are

$$u(y,0)=0,\ T(y,0)=0\ \text{for}\ y\geq 0,$$
$$\left.\frac{\partial u(y,t)}{\partial y}\right|_{y=0}=f(t),\ T(0,t)=1\ \text{for}\ t>0,\tag{11}$$
$$u(y,t),T(y,t)\to 0\ \text{as}\ y\to\infty.$$

The dimensionless temperature and the surface heat transfer rate, as it results from [1, (13) and (15)], are given by

$$T(y,t)=\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr_{eff}}}{t}}\right),$$
$$\left.\frac{\partial T(y,t)}{\partial y}\right|_{y=0}=-\sqrt{\frac{\mathrm{Pr_{eff}}}{\pi t}},\tag{12}$$

where $\operatorname{erfc}(\cdot)$ is the complementary error function of Gauss.

Applying the Laplace transform to (9) and bearing in mind the corresponding initial and boundary conditions for $u(y,t)$, we find that

$$\frac{\partial^2\bar{u}(y,q)}{\partial y^2}-(q+K_p)\,\bar{u}(y,q)$$
$$=-\frac{1}{q}\exp\left(-y\sqrt{\mathrm{Pr_{eff}}q}\right),\tag{13}$$

where the Laplace transform $\bar{u}(y,q)$ of $u(y,t)$ has to satisfy the conditions

$$\left.\frac{\partial\bar{u}(y,q)}{\partial y}\right|_{y=0}=F(q),\ \bar{u}(y,q)\to 0\ \text{as}\ y\to\infty.\tag{14}$$

Here, $F(q)$ is the Laplace transform of $f(t)$, and the solution of (13) with conditions (14) is given by

$$\bar{u}(y,q)=\frac{\exp\left(-y\sqrt{\mathrm{Pr_{eff}}q}\right)}{q\left[q(1-\mathrm{Pr_{eff}})+K_p\right]}$$
$$-\frac{\sqrt{\mathrm{Pr_{eff}}q}\exp\left(-y\sqrt{q+K_p}\right)}{q\left[q(1-\mathrm{Pr_{eff}})+K_p\right]\sqrt{q+K_p}}\tag{15}$$
$$-F(q)\frac{\exp\left(-y\sqrt{q+K_p}\right)}{\sqrt{q+K_p}}.$$

In order to obtain the (y,t)-domain solution for velocity, we firstly rewrite $\bar{u}(y,q)$ in the equivalent but suitable form

$$\bar{u}(y,t)=\frac{1}{K_p}\left[\frac{\exp\left(-y\sqrt{\mathrm{Pr_{eff}}q}\right)}{q}-\frac{\exp\left(-y\sqrt{\mathrm{Pr_{eff}}q}\right)}{q-b}\right]$$
$$+\frac{b\sqrt{\mathrm{Pr_{eff}}}}{K_p}\frac{1}{(q-b)\sqrt{q}}\frac{\exp\left(-y\sqrt{q+K_p}\right)}{\sqrt{q+K_p}}$$
$$-F(q)\frac{\exp\left(-y\sqrt{q+K_p}\right)}{\sqrt{q+K_p}},\tag{16}$$

where $b=K_p/(\mathrm{Pr_{eff}}-1)$ if $\mathrm{Pr_{eff}}\neq 1$. Applying the inverse Laplace transform to (16), the velocity $u(y,t)$ can be written as a sum, namely

$$u(y,t)=u_t(y,t)+u_m(y,t)$$
$$\text{for}\ \mathrm{Pr_{eff}}\neq 1\ \text{and}\ K_p\neq 0,\tag{17}$$

where

$$u_t(y,t)=\frac{1}{K_p}\operatorname{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr_{eff}}}{t}}\right)$$
$$-\frac{e^{bt}}{2K_p}\left[\exp\left(-y\sqrt{b\mathrm{Pr_{eff}}}\right)\operatorname{erfc}\left(\frac{y\sqrt{\mathrm{Pr_{eff}}}}{2\sqrt{t}}-\sqrt{bt}\right)\right.$$
$$\left.+\exp\left(y\sqrt{b\mathrm{Pr_{eff}}}\right)\operatorname{erfc}\left(\frac{y\sqrt{\mathrm{Pr_{eff}}}}{2\sqrt{t}}+\sqrt{bt}\right)\right]$$

$$+\frac{b\sqrt{\mathrm{Pr}_{\mathrm{eff}}}}{2K_{\mathrm{p}}\sqrt{\pi(b+K_{\mathrm{p}})}}\int_0^t \frac{\mathrm{e}^{bs}}{\sqrt{t-s}}\Bigg[\exp\left(-y\sqrt{b+K_{\mathrm{p}}}\right) \cdot\mathrm{erfc}\left(\frac{y}{2\sqrt{s}}-\sqrt{(b+K_{\mathrm{p}})s}\right)-\exp\left(y\sqrt{b+K_{\mathrm{p}}}\right) \cdot\mathrm{erfc}\left(\frac{y}{2\sqrt{s}}+\sqrt{(b+K_{\mathrm{p}})s}\right)\Bigg]\,\mathrm{d}s, \tag{18}$$

corresponds to the thermal effects, and

$$u_m(y,t)=-\frac{1}{\sqrt{\pi}}\int_0^t\frac{f(t-s)}{\sqrt{s}} \cdot\exp\left(-\frac{y^2}{4s}-K_{\mathrm{p}}s\right)\,\mathrm{d}s, \tag{19}$$

becomes zero if the shear stress on the boundary vanishes.

Direct computations show that $T(y,t)$ and $u(y,t)$ given by (12) and (17) satisfy all imposed initial and boundary conditions. In order to show that $u(y,t)$ satisfies the boundary condition $(11)_3$, for instance, let us firstly observe that direct computations imply

$$\left.\frac{\partial u_t(y,t)}{\partial y}\right|_{y=0}=0 \quad\text{and}\quad \frac{\partial u_m(y,t)}{\partial y}=\frac{y}{2\sqrt{\pi}}\int_0^t\frac{f(t-s)}{s\sqrt{s}}\exp\left(-\frac{y^2}{4s}-K_{\mathrm{p}}s\right)\mathrm{d}s. \tag{20}$$

The last equality is equivalent to

$$\frac{\partial u_m(y,t)}{\partial y}=\frac{2}{\sqrt{\pi}}\int_{\frac{y}{(2\sqrt{t})}}^{\infty} f\left(t-\frac{y^2}{4s^2}\right) \cdot\exp\left(-s^2-K_{\mathrm{p}}\frac{y^2}{4s^2}\right)\mathrm{d}s, \tag{21}$$

that clearly implies $\left.\frac{\partial u_m(y,t)}{\partial y}\right|_{y=0}=f(t)$.

Finally, let us observe that $T(y,t)$ given by (12) is valid for all positive values of $\mathrm{Pr}_{\mathrm{eff}}$ while the component $u_t(y,t)$ of the solution for velocity is not valid for $\mathrm{Pr}_{\mathrm{eff}}=1$.

Consequently, in this case, $u_t(y,t)$ has to be rederived starting again from (15). By making $\mathrm{Pr}_{\mathrm{eff}}=1$ in the first two terms of (15) and applying again the inverse Laplace transform, we find that

$$u(y,t)=\frac{1}{K_{\mathrm{p}}}\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right)-\frac{1}{\pi K_{\mathrm{p}}}\int_0^t\frac{1}{\sqrt{s(t-s)}} \cdot\exp\left(-\frac{y^2}{4s}-K_{\mathrm{p}}s\right)\mathrm{d}s-\frac{1}{\sqrt{\pi}}\int_0^t\frac{f(t-s)}{\sqrt{s}} \cdot\exp\left(-\frac{y^2}{4s}-K_{\mathrm{p}}s\right)\mathrm{d}s \quad\text{for } K_{\mathrm{p}}\neq 0. \tag{22}$$

4. Limiting Cases

In the following, for completion, let us consider some limiting cases of general solutions.

4.1. Solution in the Absence of Thermal Radiation ($N_{\mathrm{r}}\to 0$)

In the absence of thermal radiation, namely in the pure convection, the corresponding solutions can directly be obtained from general solutions by making $N_{\mathrm{r}}\to 0$ and therefore substituting the effective Prandtl number $\mathrm{Pr}_{\mathrm{eff}}$ by the Prandtl number Pr. The dimensionless temperature $T(y,t)$ and the surface heat transfer rate, for instance, take the simplified forms

$$T(y,t)=\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}}{t}}\right) \quad\text{and}\quad \left.\frac{\partial T(y,t)}{\partial y}\right|_{y=0}=-\sqrt{\frac{\mathrm{Pr}}{\pi t}}. \tag{23}$$

4.2. Solution in the Absence of Mechanical Effects

Let us now assume that the infinite plate is kept at rest all the time. In this case, the function $f(t)$ is zero for each real value of t and the component $u_m(y,t)$ of velocity is identically zero. Consequently, the velocity of the fluid $u(y,t)$ reduces to the thermal component $u_t(y,t)$ given by (18). Its temperature, as well as the surface heat transfer rate, is given by the same equality (12).

4.3. Solution in the Absence of Porous Effects ($K_{\mathrm{p}}\to 0$)

The temperature distribution in the fluid mass, as it results from (12), is not affected by the porosity of medium, and the velocity corresponding to the purely fluid regime, i. e. infinite permeability, cannot be obtained from general solution (17) by making $K_{\mathrm{p}} \to 0$. So, we must start again from (15). For $K_{\mathrm{p}} = 0$ this equality becomes

$$\bar{u}(y,q) = \frac{1}{1-\mathrm{Pr}_{\mathrm{eff}}} \cdot \left[\frac{\exp\left(-y\sqrt{\mathrm{Pr}_{\mathrm{eff}} q}\right)}{q^2} - \sqrt{\mathrm{Pr}_{\mathrm{eff}}}\frac{\exp\left(-y\sqrt{q}\right)}{q^2}\right] - F(q)\frac{\exp\left(-y\sqrt{q}\right)}{\sqrt{q}}, \tag{24}$$

and the velocity of the fluid is given by

$$u(y,t) = \frac{1}{1-\mathrm{Pr}_{\mathrm{eff}}}\left[\left(t+\frac{y^2}{2}\mathrm{Pr}_{\mathrm{eff}}\right)\mathrm{erfc}\left(\frac{y}{2}\sqrt{\frac{\mathrm{Pr}_{\mathrm{eff}}}{t}}\right) - y\sqrt{\frac{\mathrm{Pr}_{\mathrm{eff}} t}{\pi}}\exp\left(-\frac{y^2}{4t}\mathrm{Pr}_{\mathrm{eff}}\right)\right] - \frac{\sqrt{\mathrm{Pr}_{\mathrm{eff}}}}{1-\mathrm{Pr}_{\mathrm{eff}}}\left[\left(t+\frac{y^2}{2}\right) \cdot \mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right) - y\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right)\right] - \frac{1}{\sqrt{\pi}}\int_0^t \frac{f(t-s)}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s}\right)\mathrm{d}s \text{ if } \mathrm{Pr}_{\mathrm{eff}} \neq 1. \tag{25}$$

Furthermore, if $\mathrm{Pr}_{\mathrm{eff}} = 1$, the solution of our problem is (see (13) and (14))

$$\bar{u}(y,q) = \frac{1}{2q}\left(\frac{1}{q}+\frac{y}{\sqrt{q}}\right)\exp\left(-y\sqrt{q}\right) - F(q)\frac{\exp\left(-y\sqrt{q}\right)}{\sqrt{q}}, \tag{26}$$

and the corresponding velocity has the simplified form

$$u(y,t) = \frac{1}{2}\left[\left(t-\frac{y^2}{2}\right)\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right) + y\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right)\right] - \frac{1}{\sqrt{\pi}} \cdot \int_0^t \frac{f(t-s)}{\sqrt{s}}\exp\left(-\frac{y^2}{4s}\right)\mathrm{d}s. \tag{27}$$

5. Special Cases

In order to underline the theoretical value of the general solution (17) for velocity, as well as to gain physical insight of the flow regime, we consider some special cases whose technical relevance is well known in the literature.

5.1. Case $f(t) = fH(t)$

Let us firstly consider $f(t) = fH(t)$ where f is a dimensionless constant and $H(\cdot)$ is the unit Heaviside step function. In this case, after time $t = 0$, the infinite plate applies a constant shear to the fluid. The thermal component of velocity $u_t(y,t)$ remain unchanged, while $u_m(y,t)$ takes the simplified form

$$u_{m0}(y,t) = -\frac{f}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{s}}\exp\left(-\frac{y^2}{4s} - K_{\mathrm{p}}s\right)\mathrm{d}s, \tag{28}$$

or equivalently

$$u_{m0}(y,t) = -\frac{f}{K_{\mathrm{p}}}\exp\left(-K_{\mathrm{p}}y\right) + \frac{2f}{\sqrt{\pi}}\int_{\sqrt{t}}^{\infty} \cdot \exp\left(-\frac{y^2}{4s^2} - K_{\mathrm{p}}s^2\right)\mathrm{d}s \text{ if } K_{\mathrm{p}} \neq 0. \tag{29}$$

In the case $K_{\mathrm{p}} = 0$, (28) takes the simplified form (in agreement with [14, (23)]

$$u_{m0}(y,t) = -\frac{f}{\sqrt{\pi}}\int_0^t \frac{1}{\sqrt{s}}\exp\left(-\frac{y^2}{4s}\right)\mathrm{d}s, \tag{30}$$

or evaluating the integral,

$$u_{m0}(y,t) = fy\,\mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right) - 2f\sqrt{\frac{t}{\pi}}\exp\left(-\frac{y^2}{4t}\right). \tag{31}$$

In conclusion, the velocity field corresponding to the case when the plate applies a constant shear to the fluid is given by (17) where $u_t(y,t)$ is defined by one of (18), (22), or (27), and $u_m(y,t)$ is given by the equalities (28) or (31).

5.2. Case $f(t) = ft^a$ $(a > 0)$

Introducing $f(t) = ft^a$ into (19), we find that

$$u_{\mathrm{m}a}(y,t) = -\frac{f}{\sqrt{\pi}}\int_0^t \frac{(t-s)^a}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s} - K_{\mathrm{p}}s\right)\mathrm{d}s. \tag{32}$$

Expression of the mechanical component of velocity corresponding to $K_{\mathrm{p}} = 0$, namely

$$u_{\mathrm{m}a}(y,t) = -\frac{f}{\sqrt{\pi}}\int_0^t \frac{(t-s)^a}{\sqrt{s}}\exp\left(-\frac{y^2}{4s}\right)\mathrm{d}s, \tag{33}$$

is equivalent to [12, (4.1)] with $\alpha = 0$. This motion, unlike those corresponding to the cases 5.1 and 5.3, is unsteady and remain unsteady all the time.

Of interest is the case $a = 1$ when the plate applies a constantly accelerating shear stress to the fluid. The corresponding expression of the mechanical component $u_{m1}(y,t)$, resulting from (32), is

$$u_{m1}(y,t) = -\frac{f}{\sqrt{\pi}} \int_0^t \frac{t-s}{\sqrt{s}} \exp\left(-\frac{y^2}{4s} - K_\mathrm{p} s\right) \mathrm{d}s = \int_0^t u_{m0}(y,s)\,\mathrm{d}s\,. \tag{34}$$

5.3. Case $f(t) = f\sin(\omega t)$

By now letting $f(t) = f\sin(\omega t)$ in the general expression (19) of $u_m(y,t)$, it results that

$$u_m(y,t) = -\frac{f}{\sqrt{\pi}} \int_0^t \frac{\sin[\omega(t-s)]}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s} - K_\mathrm{p} s\right) \mathrm{d}s\,. \tag{35}$$

This is the mechanical component of the fluid velocity in the motion induced by an infinite plate that applies an oscillating shear stress to the fluid. It can be written as a sum between steady-state and transient solutions:

$$u_\mathrm{ms}(y,t) = -\frac{f}{\sqrt{\pi}} \int_0^\infty \frac{\sin[\omega(t-s)]}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s} - K_\mathrm{p} s\right) \mathrm{d}s\,, \tag{36}$$

$$u_\mathrm{mt}(y,t) = \frac{f}{\sqrt{\pi}} \int_t^\infty \frac{\sin[\omega(t-s)]}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s} - K_\mathrm{p} s\right) \mathrm{d}s\,. \tag{37}$$

In the above relations, ω is the dimensionless frequency of the shear stress. In the absence of porosity, the steady-state component

$$u_\mathrm{ms}(y,t) = -\frac{f}{\sqrt{\pi}} \int_0^\infty \frac{\sin[\omega(t-s)]}{\sqrt{s}} \cdot \exp\left(-\frac{y^2}{4s}\right) \mathrm{d}s \tag{38}$$

can be written in the simplified form

$$u_\mathrm{ms}(y,t) = \frac{f}{\sqrt{\omega}} \exp\left(-y\sqrt{\frac{\omega}{2}}\right) \cdot \cos\left(\omega t - y\sqrt{\frac{\omega}{2}} + \frac{\pi}{4}\right). \tag{39}$$

For a check of results, let us determine the steady shear stress component corresponding to the steady-state velocity (39), namely [13, (24)],

$$\tau_\mathrm{ms}(y,t) = f\exp\left(-y\sqrt{\frac{\omega}{2}}\right) \cdot \sin\left(\omega t - y\sqrt{\frac{\omega}{2}}\right). \tag{40}$$

As expected, it is in accordance with the dimensional form resulting from [14, (30)].

6. Numerical Results and Discussion

In order to study the behaviour of dimensionless velocity and temperature fields and to get some physical insight of the obtained results, a series of numerical calculations was carried out for different values of pertinent parameters that describe the flow characteristics. All graphs correspond to the case when the plate applies a constant shear stress to the fluid. Figure 1 exhibits the dimensionless velocity profiles at different times and fixed values of the material parameters Pr_eff and K_p and the shear f on the boundary. As expected, the velocity of the fluid increases in time and smoothly decreases to zero for y going to infinity. Figure 2 shows the influence of the effective Prandtl number on velocity. The velocity of the fluid is a decreasing function with respect to Pr_eff. This result agrees well with that resulting from [1, Fig. 3] because Pr_eff decreases if the radiation–conduction parameter N_r increases.

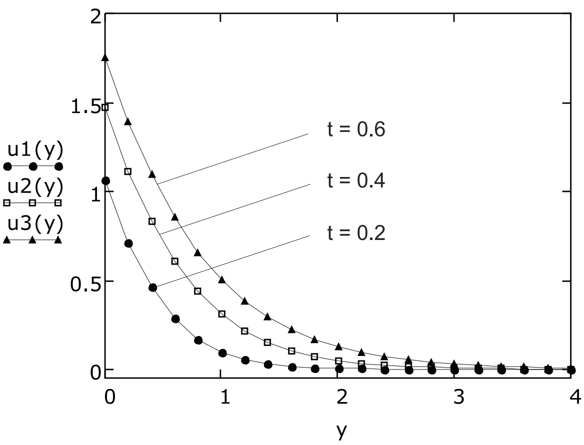

Fig. 1. Non-dimensional velocity profiles for $\mathrm{Pr}_\mathrm{eff} = 0.35$ $(N_\mathrm{r} = 1, \mathrm{Pr} = 0.7)$, $K_\mathrm{p} = 1$, and different values of t when the plate applies a constant shear stress $f = -2$ to the fluid.

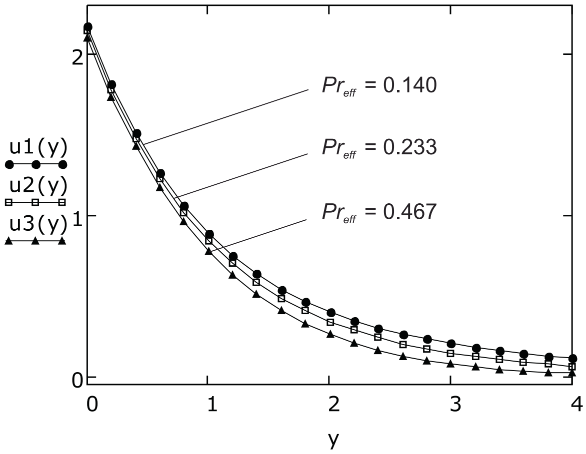

Fig. 2. Non-dimensional velocity profiles for $t = 1$, $K_p = 1$, and different values of $\mathrm{Pr}_{\mathrm{eff}}$ when the plate applies a constant shear stress $f = -2$ to the fluid.

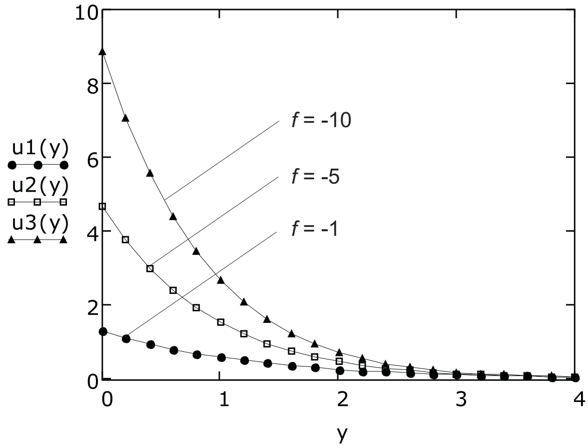

Fig. 4. Non-dimensional velocity profiles for $t = 1$, $\mathrm{Pr}_{\mathrm{eff}} = 0.35$, $K_p = 1$, and different values of constant shear stress f.

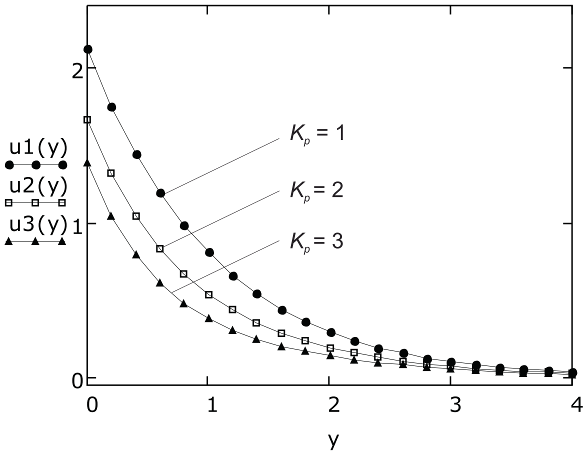

Fig. 3. Non-dimensional velocity profiles for $t = 1$, $\mathrm{Pr}_{\mathrm{eff}} = 0.35$, and different values of K_p when the plate applies a constant shear stress $f = -2$ to the fluid.

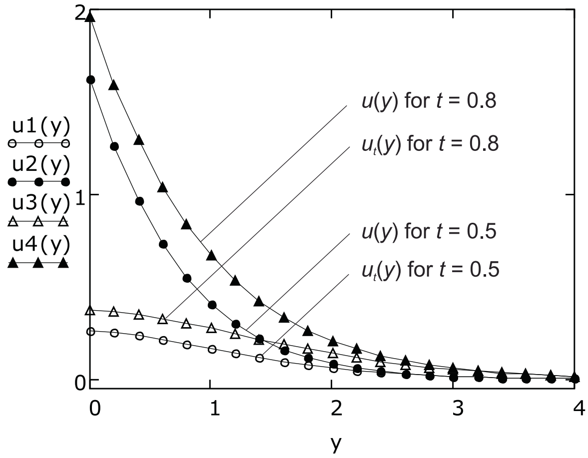

Fig. 5. Comparison between dimensionless velocity $u(y,t)$ and its thermal component $u_t(y,t)$ for $\mathrm{Pr}_{\mathrm{eff}} = 0.35$, $K_p = 1$, $f = -2$, and $t = 0.5$ and 0.8.

The effects of permeability parameter K_p on the spatial distribution of the dimensionless velocity are presented in Figure 3. The inverse permeability parameter K_p, as defined by (8), is inverse proportional to the permeability of the medium. The resistance of porous medium increases if its permeability decreases. Consequently, the velocity of the fluid decreases with respect to K_p. However, this change of velocity is maximum near the plate, decreases with respect to y, and finally approach to zero. The profiles of velocity monotonically decay for all values of K_p, and the boundary layer thickness decreases when K_p increases. The spatial variation of the dimensionless velocity $u(y,t)$ with the shear stress f induced by the boundary plate is plotted against y in Figure 4. As expected, the velocity of the fluid decreases for increasing values of f (by negative values) and this result is in accordance with that of Erdogan [21, Fig. 3]. The influence of thermal effects on the fluid motion is shown by Figure 5 where the dimensionless velocity $u(y,t)$ against y is compared with its thermal component $u_t(y,t)$. As expected, the mechanical effects are stronger but the thermal influence on velocity is also significant.

Expressions of the dimensionless temperature and surface heat transfer rate, as we previously specified,

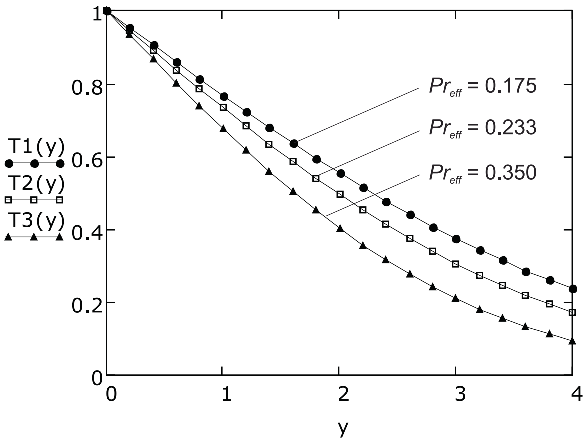

Fig. 6. Dimensionless temperature profiles for $t = 1$ and different values of $\mathrm{Pr}_{\mathrm{eff}}$.

are identical to those from [1, (13) and (15)]. Consequently, there is no reason to present again their variations with respect to time, Prandtl number or radiation–conduction parameter. Of interest seems to be here their variations against y for different values of the effective Prandtl number. Such a variation for temperature is presented in Figure 6 for $t = 1$. It is clearly observed that an increase of the effective Prandtl number $\mathrm{Pr}_{\mathrm{eff}}$ implies a significant decrease of the temperature throughout the fluid. The temperature of the fluid, for different values of $\mathrm{Pr}_{\mathrm{eff}}$, smoothly decreases from a maximum at the boundary to a minimum value for large values of y. Further, the values of $T(y,t)$ at any distance y from the plate are always higher for $\mathrm{Pr}_{\mathrm{eff}} = 0.175$ than those for $\mathrm{Pr}_{\mathrm{eff}} = 0.233$ or 0.350. The thermal boundary layer thickness also decreases for increasing $\mathrm{Pr}_{\mathrm{eff}}$.

7. Conclusions

Heat transfer and the motion of a viscous fluid over a heated infinite plate that applies an arbitrary shear stress $f(t)$ to the fluid are analytically studied. Radiative and porous effects are taken into consideration and exact solutions for the dimensionless velocity and temperature are obtained by means of Laplace transforms. These solutions, presented in simple forms in terms of the complementary error function of Gauss, satisfy both governing equations and all imposed initial and boundary conditions. The dimensionless temperature depends on $\mathrm{Pr}_{\mathrm{eff}}$ only, and the fluid velocity is presented as a sum of thermal and mechanical components. All results regarding velocity are new and its mechanical component reduces to known forms from the literature in absence of porous effects.

Some significant limiting cases, excepting those corresponding to $\mathrm{Pr}_{\mathrm{eff}} = 1$ and $K_{\mathrm{p}} = 0$ whose solutions are separately established, are easy obtained from general solutions. In all cases, the temperature of the fluid does not depend on porosity and shear stress on the boundary. This is possible as the viscous dissipation is not taken into consideration. Further, as expected, both components of velocity are affected by the porosity of the medium and the number of essential parameters is reduced by a suitable selection of the reference velocity U.

Finally, in order to underline some physical insight of present results, three special cases of technical relevance motions are considered. The first case corresponds to the fluid motion due to an infinite plate that applies a constant shear to the fluid. Figures 1 – 4 are prepared to bring to light the effects of pertinent parameters on the velocity field. A comparison between the dimensionless velocity $u(y,t)$ and its thermal component $u_t(y,t)$ is presented in Figure 5. It is clearly seen that the thermal effects, as well as the mechanical ones, have a significant influence on the fluid motion. The main findings are:

(i) The dimensionless temperature, as well as the surface heat transfer rate, is not influenced by the porosity of the medium and the shear stress on the boundary. It depends only on the effective Prandtl number $\mathrm{Pr}_{\mathrm{eff}}$.

(ii) The dimensionless velocity is presented as a sum of thermal and mechanical components. The influence of thermal effects on velocity is also significant.

(iii) The velocity of the fluid is a decreasing function with respect to $\mathrm{Pr}_{\mathrm{eff}}$, K_{p}, and f.

(iv) The boundary layer thickness, as well as the thermal boundary layer thickness, decreases when the effective Prandtl number increases.

Acknowledgement

The author Mehwish Rana is highly thankful to the Abdus Salam School of Mathematical Sciences, GC University, Lahore and Higher Education Commision of Pakistan for generous supporting and facilitating her research work.

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