Optimal Homotopy Perturbation Method for a Non-Conservative Dynamical System of a Rotating Electrical Machine

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A version of the optimal homotopy perturbation method (OHPM) is applied in this study to derive highly accurate analytical expressions for the solutions to a non-conservative dynamical system of a rotating electrical machine. The main advantage of this procedure consists of providing us with a convenient and rigorous way to control the approximate solutions by means of some initially unknown parameters which are optimally determined later. Comparisons with numerical results reveal an excellent agreement, which demonstrates the effectiveness of the proposed method in analyzing non-conservative oscillators.

Key words: Optimal Homotopy Perturbation Method (OHPM) ; Rotating Electrical Machine; Differential Equation with Variable Coefficients.

1. Introduction

Rotating electrical machines are complex engineering systems which combine electrical and mechanical concepts. From mechanical point of view, these systems inherently involve the presence of a rotor, which often raises some dynamical problems typical for rotor dynamics [1, 2]. The most common dynamic problem encountered by these systems is generated by unbalanced forces; sometimes even a small amount of unbalance can cause vibration that can reach undesirably high values, very detrimental for a properly work of the mechanical system. It is known that no absolutely perfect balancing can be realized and therefore there will always be a residual unbalance. Other dynamical problems could be also generated by bad or nonlinear bearings, mechanical looseness, misalignment, and even some electrical problems.

From engineering point of view it is very important to make some reliable tools available intended to predict, analyze, and correct these problems in order to obtain higher speed machines, to prevent unexpected failures or to assure longer periods between downtimes. Obviously, when an improvement of dynamic characteristics of these systems is needed, analytical developments and numerical simulations are preferable to experimental investigations because of lower costs.

Concerning analytical approaches, classical analytical methods often fail because of the complexity and strong nonlinearity of these systems. That is why scientists are continuously concerned in developing new analytical methods, valid for these strongly nonlinear systems, some of them being non-conservative systems. Analytical techniques intended to solve nonlinear problems or some problems described by differential equations with variable coefficients have been dominated by the perturbation methods [3]. However, perturbation methods have their limitations, so scientists search for new analytical methods valid for a large class of nonlinear problems.

In recent years, a growing interest towards the application of the homotopy technique in nonlinear problems has appeared. In 1992, Liao [4] presented his homotopy analysis method (HAM). He is the first who introduced an unknown parameter h in the homotopy equation providing a way to ensure the convergence of analytic approximations. Later, homotopy perturbation method (HPM) has been developing very fast, and various modifications appeared recently, among which the version coupled with the least squares technology has been caught much attention. The HPM admits some unknown parameters in the obtained series solutions, which can be identified after few iteration steps using the method of least squares. A similar solution procedure was employed in 2007 by Marinca et al. [5] proposing a new homotopy approach, namely the optimal homotopy asymptotic method (OHAM), which proved to be another reliable approach to nonlinear problems, selecting the optimal values of some convergence-control parameters. The basic idea of the optimal homotopy perturbation method and the optimal homotopy asymptotic method has been directly used later in [6] resulting in a so-called optimal homotopy analysis approach.

In this paper, an application of a modified version of the homotopy perturbation method is presented, which proposes a solution procedure simpler than that suggested in [5]. The rotating electrical machine under study is modelled as a non-conservative single-degree-of-freedom dynamic system subject to a parametric excitation caused by an axial thrust and a forcing excitation caused by an unbalanced force of the rotor. In these conditions, the system can be described by the following second-order differential equation with variable coefficients:

$$m\ddot{u}+c\dot{u}+k(1-\lambda\sin(\omega_2 t))u = f\sin(\omega_1 t) \quad (1)$$

subject to the initial conditions

$$u(0)=A\,,\;\; \dot{u}(0)=0\,, \quad (2)$$

where m is the lump mass of the rotor, c the linear damping coefficient, k the shaft stiffness, ω_1 and ω_2 are the forcing and the parametric frequencies, respectively, and f and $k\lambda$ the amplitudes of forcing and parametric excitations, respectively. Since usual analytic methods cannot be employed to solve (1), usually numerical methods are resorted to in order to obtain a solution. For such kind of damped non-conservative oscillation systems, Lim et al. [7] proposed an approach combining the harmonic balance method and the method of averaging to obtain accurate solutions.

In what follows, we propose one of the newest analytical techniques, namely the optimal homotopy perturbation method (OHPM) [8, 9] to investigate the above described dynamical system. This version of OHPM distinguishes by a specific construction of the homotopy and a special construction of the auxiliary functions involving some convergence-control parameters. Different from other traditional methods, the validity of OHPM is independent on whether or not there exist small parameters in the considered differential equation. This procedure provides us with a convenient way to optimally control the approximate solution.

2. Basic Ideas of OHPM

To explain the basic ideas of OHPM for solving differential equations with variable coefficients or nonlinear differential equations, we consider the equation

$$L(u(r))+V(u(r))=0,\;\; r\in D\,, \quad (3)$$

subject to the boundary conditions

$$B\left(u,\frac{\partial u}{\partial r}\right)=0,\;\; r\in\Gamma\,, \quad (4)$$

where L is a linear operator, V an operator with variable coefficients, B a boundary operator, Γ is the boundary of the domain D, and $\partial/\partial n$ denotes differentiation along the normal drawn outwards from the domain D.

We construct a homotopy $u(r,p): D\times[0,1]\to\mathbb{R}$ which satisfies

$$H(v,p)=L(v)+pV(v)=0\,, \quad (5)$$

where $p\in[0,1]$ is an embedding parameter. From (5) it follows that

$$H(v,0)=L(v)=0\,, \quad (6)$$
$$H(v,1)=L(v)+V(v)=0\,. \quad (7)$$

Thus, the changing process of p from zero to unity is just that of $u(r,p)$ from v_0 given by (6) to $v(r)$ given by (7). In topology, this is called deformation, and $L(v)$ and $L(v)+V(v)$ are called homotopic. So, it is very right to assume that the solutions of (6) and (7) can be expressed as

$$v=v_0+pv_1+p^2v_2+\ldots\,. \quad (8)$$

The exact solution of (3) can be readily obtained as

$$\begin{aligned} u&=\lim_{p\to 1}(v_0+pv_1+p^2v_2+\ldots)\\ &=v_0+v_1+v_2+\ldots\,. \end{aligned} \quad (9)$$

These are the basic ideas of the homotopy perturbation method.

Generally speaking, the operator $V(u)$ depends on u, and therefore one can write

$$V(u) = F(t,u). \tag{10}$$

Applying the Taylor series theorem for the real value α, we obtain

$$F(t,u+\alpha) = F(t,u) + \frac{\alpha}{1!}F_u(t,u) + \dots, \tag{11}$$

where $F_u = \frac{\partial F}{\partial u}$.

At this moment, we introduce a new homotopy and the unknown auxiliary functions $K_{1i}(t,C_k)$, $i = 1,2,\dots,s$, that depend on the variable t and some convergence-control parameters $C_1, C_2, \dots$ such that

$$\begin{aligned} H(v,p) = L(v) + p[K_{11}(t,C_k)F(t,v_0) \\ + K_{12}(t,C_k)F_v(t,v_0)] = 0. \end{aligned} \tag{12}$$

A whole set of equations is obtained by equating the coefficients of like powers of p for (3). More precisely, we have the following two equations:

$$L(v_0) = 0, \;\; B(v_0,\dot{v}_0) = 0, \tag{13}$$

$$\begin{aligned} &L(v_1) + K_{11}(t,C_k)F(t,v_0) + K_{12}(t,C_k)F_v(t,v_0) \\ &= 0, \;\; B(v_1,\dot{v}_1) = 0. \end{aligned} \tag{14}$$

It is worthy to note that the functions K_{1i}, $i = 1,2$, are not unique and can be chosen such that the products $K_{1i}F_a$ and F_a are of the same form. In this way, using this alternative of OHPM, we expect that only one iteration is needed to achieve accurate solutions. The convergence-control parameters C_k, $k = 1,2,\dots,s$, which appear in the expression of the functions $K_{1i}(t,C_k)$ can be optimally determined. This can be done via various methods, such as the least square method, the collocation method, the Galerkin method, and so on. For example, these parameters can be optimally determined by imposing the residual functional

$$J(C_1,C_2,\dots,C_s) = \int_a^b [L(\bar{v} + V(\bar{v})]^2 \,\mathrm{d}r \tag{15}$$

being minimum, which leads to the system of algebraic equations

$$\frac{\partial J}{\partial C_i} = 0, \;\; i = 1,2,\dots,s, \tag{16}$$

where a and b are two values from the domain of interest, and $\bar{v}$ is the first-order approximate solution, obtained from (9):

$$\bar{v} = v_0 + v_1. \tag{17}$$

Another convenient procedure to optimally determine the convergence-control parameters C_k employs the residual R obtained by substituting the approximate analytical solution depending on unknown parameters C_k in the initial equation. For example, using a collocation-type procedure: if $t_i \in [a,b]$, $i = 1,2,\dots,s$, we use the residual R given by

$$R(t,C_1,C_2,\dots,C_s) = L(\bar{v}) + V(\bar{v}) \tag{18}$$

and obtain a system of algebraic equations

$$\begin{aligned} &R(t,C_k) = R(t_2,C_k) = \dots = R(t_s,C_k) = 0, \\ &k = 1,2,\dots,s. \end{aligned} \tag{19}$$

The solutions of (3) subject to the initial conditions given by (4) can immediately be determined when the convergence-control parameters C_k are made available.

3. Examples

The validity of the proposed approach is illustrated on the dynamical system of a rotating electrical machine whose motion is described by (1) and (2). Equation (1) can be written in the dimensionless form

$$\ddot{u} + 2\mu\dot{u} + \omega^2 u - \alpha u \sin(\omega_2 t) - \beta \sin(\omega_1 t) = 0, \tag{20}$$

where

$$\mu = \frac{c}{2m}, \;\; \omega^2 = \frac{k}{m}, \;\; \alpha = \frac{k\lambda}{m}, \;\; \beta = \frac{f}{m}. \tag{21}$$

In accordance with (3) and (20), the linear operator is chosen as

$$L(u) = \ddot{u} + 2\mu\dot{u} + \omega^2 u \tag{22}$$

and the operator with variable coefficients as

$$V(u) = -\alpha u \sin\omega_2 t - \beta \sin\omega_1 t. \tag{23}$$

The initial approximation $v_0(t)$ is obtained from (13):

$$\ddot{v}_0 + 2\mu\dot{v}_0 + \omega^2 v_0 = 0 \tag{24}$$

with initial conditions

$$v_0(0) = A, \;\; \dot{v}_0(0) = 0. \tag{25}$$

The solution of (24) and (25) is

$$v_0(t) = \left(A\cos\Omega t + \frac{\mu A}{\Omega}\sin\Omega t\right)\exp(-\mu t), \tag{26}$$

where $\Omega = \sqrt{\omega^2 - \mu^2}$.

Equation (14) becomes

$$\ddot{v}_1 + 2\mu\dot{v}_1 + \omega^2 v_1 + K_{11}(t,C_k)(-\alpha v_0 \sin\omega_2 t - \beta\sin\omega_1 t) + K_{12}(t,C_k)(-\alpha\sin\omega_2 t) = 0\,, \quad v_1(0)=0\,,\ \dot{v}_1(0)=0\,. \tag{27}$$

Generally, for this kind of damped oscillators, only the transitory behaviour is worthy of interest, because at infinity, the motion becomes a harmonic one. In this case, if only transitory behaviour is studied, we choose the functions $K_{11}(t,C_k)$ and $K_{12}(t,C_k)$ in the form

$$K_{11}(t,C_k) = C_1\exp(-\mu t)\,, \tag{28}$$

$$K_{12}(t,C_k) = (C_2 + 2C_3\cos\Omega t + 2C_4\sin\Omega t)\cdot\exp(-\mu t)\,. \tag{29}$$

As we mentioned before, choosing $K_1(t,C_k)$ is not unique. We can choose for example

$$K_{11}(t,C_k) = (C_1 + 2C_2\cos\Omega t + 2C_3\sin\Omega t)\cdot\exp(-\mu t)\,, \tag{30}$$

$$K_{12}(t,C_k) = (C_4 + 2C_5\cos\Omega t + 2C_6\sin\Omega t)\cdot\exp(-\mu t)\,, \tag{31}$$

and so on.

The first-order approximate solution of (20) is obtained from (17), (26), (27), and (29) in the form

$$\begin{aligned}\bar{v}(t) = \Bigg[&(A+a)\cos\Omega t + \left(\frac{\mu A}{\Omega}+b\right)\sin\Omega t + \frac{\beta C_1}{\Omega^2-\omega_1^2}\sin\omega_1 t + \frac{\alpha C_2}{\Omega^2-\omega_2^2}\sin\omega_2 t - \frac{\alpha C_3}{\omega_2(2\Omega+\omega_2)}\sin(\Omega+\omega_2)t\\
&- \frac{\alpha C_3}{\omega_2(2\Omega-\omega_2)}\sin(\Omega-\omega_2)t + \frac{\alpha C_4}{\omega_2(2\Omega-\omega_2)}\cos(\Omega-\omega_2)t + \frac{\alpha C_4}{\omega_2(2\Omega+\omega_2)}\cos(\Omega+\omega_2)t\Bigg]\exp(-\mu t)\\
&+ \frac{1}{2}\alpha C_1\left\{\frac{\frac{\mu A}{\Omega}[2\Omega(\Omega+\omega_2)-\omega^2+(\Omega+\omega_2)^2]\cos(\Omega+\omega_2)t + A[\omega^2-(\Omega+\omega_2)^2+\frac{2\mu^2}{\Omega}(\Omega+\omega_2)]\sin(\Omega+\omega_2)t}{[\omega^2-(\Omega+\omega_2)^2]^2+4\mu^2(\Omega+\omega_2)^2}\right.\\
&\left.+ \frac{\frac{\mu A}{\Omega}[\omega^2-2\Omega(\Omega-\omega_2)-(\Omega-\omega_2)^2]\cos(\Omega-\omega_2)t - A[\omega^2-(\Omega-\omega_2)^2+\frac{2\mu^2}{\Omega}(\Omega-\omega_2)]\sin(\Omega-\omega_2)t}{[\omega^2-(\Omega-\omega_2)^2]^2+4\mu^2(\Omega-\omega_2)^2}\right\}\\
&\cdot\exp(-2\mu t)\,,\end{aligned} \tag{32}$$

where

$$\begin{aligned}a = &-\frac{4\alpha\Omega C_4}{\omega_2(4\Omega^2-\omega_2^2)} - \frac{\alpha\mu A C_1}{2\Omega}\\
&\cdot\left\{\frac{2\Omega(\Omega+\omega_2)-\omega^2+(\Omega+\omega_2)^2}{[\omega^2-(\Omega+\omega_2)^2]^2+4\mu^2(\Omega+\omega_2)^2}\right.\\
&\left.-\frac{2\Omega(\Omega-\omega_2)-\omega^2+(\Omega-\omega_2)^2}{[\omega^2-(\Omega-\omega_2)^2]^2+4\mu^2(\Omega-\omega_2)^2}\right\}\,,\end{aligned} \tag{33}$$

$$\begin{aligned}b = &\frac{\mu a}{\Omega} - \frac{\beta\omega_1 C_1}{\Omega(\Omega^2-\omega_1^2)} - \frac{\alpha\omega_2 C_2}{\Omega(\Omega^2-\omega_2^2)}\\
&+\frac{2\alpha(2\Omega^2-\omega_2^2)C_3}{\Omega\omega_2(4\Omega^2-\omega_2^2)} + \frac{4\mu\alpha C_4}{\omega_2(4\Omega^2-\omega_2^2)} + \frac{\alpha A C_1}{2\Omega}\\
&\cdot\left\{\frac{(\Omega-\omega_2)[\omega^2-(\Omega-\omega_2)^2]-4\mu^2(\Omega-\omega_2)+\frac{2\mu^2\omega^2}{\Omega}}{[\omega^2-(\Omega-\omega_2)^2]^2+4\mu^2(\Omega-\omega_2)^2}\right.\\
&\left.-\frac{(\Omega+\omega_2)[\omega^2-(\Omega+\omega_2)^2]-4\mu^2(\Omega+\omega_2)+\frac{2\mu^2\omega^2}{\Omega}}{[\omega^2-(\Omega+\omega_2)^2]^2+4\mu^2(\Omega+\omega_2)^2}\right\}\,.\end{aligned} \tag{34}$$

It can be shown that this solution is valid only for transitory behaviour of this oscillator. If one search for uniformly valid solutions, even for $t\to\infty$, the auxiliary functions (28) and (29) should involve an additional convergence-control constant C_5 and should read

$$K'_{11}(t,C_k) = C_1\exp(-\mu t) + C_5\,, \tag{35}$$

$$K'_{12}(t,C_k) = (C_2 + 2C_3\cos\Omega t + 2C_4\sin\Omega t)\cdot\exp(-\mu t) - v_0 C_5\,. \tag{36}$$

In this case from (27), (33), and (34), we obtain

$$v'_1(t) = v_1(t) + \frac{2\mu\beta\omega_1 C_5}{(\omega^2-\omega_1^2)^2+4\mu^2\omega_1^2}\cos\omega_1 t + \frac{\beta(\omega^2-\omega_1^2)C_5}{(\omega^2-\omega_1^2)^2+4\mu^2\omega_1^2}\sin\omega_1 t\,, \tag{37}$$

where a and b are replaced by a' and b' given by

$$a' = a - \frac{2\mu\beta\omega_1 C_5}{(\omega-\omega_1^2)^2+4\mu^2\omega_1^2}\,,\qquad b' = b - \frac{(\omega^2-\omega_1^2)\beta C_5}{\left[(\omega-\omega_1^2)^2+4\mu^2\omega_1^2\right]}\,. \tag{38}$$

Therefore, the first-order approximate solution given by OHPM in this case is

$$\bar{u}_{\mathrm{OHPM}}(t) = \bar{v}(t) + \frac{2\mu\beta\omega_1 C_5}{(\omega^2-\omega_1^2)+4\mu^2\omega_1^2}\cos\omega_1 t + \frac{\beta(\omega^2-\omega_1^2)C_5}{(\omega^2-\omega_1^2)+4\mu^2\omega_1^2}\sin\omega_1 t\,. \tag{39}$$

For comparison purposes, we can obtain a solution of the same problem using the classical homotopy perturbation method (HPM). Therefore, applying HPM given by (5) and (8), a set of two equations is obtained by equating the coefficients of like powers of p in the form

$$\ddot{v}_0 + 2\mu\dot{v}_0 + \omega^2 v_0 = 0\,,\quad v_0(0) = A\,,\ \dot{v}_0(0) = 0\,, \tag{40}$$

$$\ddot{v}_1 + 2\mu\dot{v}_1 + \omega^2 v_1 - \alpha v_0\sin\omega_2 t - \beta\sin\omega_1 t = 0\,,\quad v_1(0) = \dot{v}_1(0) = 0\,. \tag{41}$$

From (40), we obtain

$$v_0(t) = \left(A\cos\Omega t + \frac{\mu A}{\Omega}\sin\Omega t\right)\exp(-\mu t)\,. \tag{42}$$

From (36) and (17), we obtain the first-order approximate solution of (20) as

$$\begin{aligned}\bar{u}_{\mathrm{HPM}}(t) &= v_0(t) + v_1(t) = \Bigg[(A+c)\cos\Omega t + \left(\frac{\mu A}{\Omega}+s\right)\\ &\cdot\sin\Omega t + \frac{\alpha\mu A}{2\Omega\omega_2(2\Omega+\omega_2)}\cos(\Omega+\omega_2)t\\ &- \frac{\alpha A}{2\omega_2(2\Omega+\omega_2)}\sin(\Omega+\omega_2)t + \frac{\alpha\mu A}{2\Omega\omega_2(2\Omega-\omega_2)}\\ &\cdot\cos(\Omega-\omega_2)t - \frac{\alpha A}{2\omega_2(2\Omega-\omega_2)}\sin(\Omega-\omega_2)\Bigg]\\ &\cdot\exp(-\mu t) - \frac{2\mu\beta\omega_1}{(\omega^2-\omega_1^2)^2+4\mu^2\omega_1^2}\cos\omega_1 t\\ &+ \frac{\beta(\omega^2-\omega_1^2)}{(\omega^2-\omega_1^2)^2+4\mu^2\omega_1^2}\sin\omega_1 t\,,\end{aligned} \tag{43}$$

where

$$c = \frac{2\mu\beta\omega_1}{(\omega-\omega_1^2)+4\mu^2\omega_1^2} - \frac{2\alpha\mu A}{\omega_2(4\Omega^2-\omega_2^2)}\,,\qquad s = \frac{\alpha(2\Omega^2-\omega_2^2)A}{\Omega\omega_2(4\Omega^2-\omega_2^2)} + \frac{\beta\omega_1(2\mu^2+\omega_1^2-\omega^2)}{\Omega\left[(\omega^2-\omega_1^2)^2+4\mu^2\omega_1^2\right]}\,. \tag{44}$$

4. Results and Discussions

In order to prove the efficiency and accuracy of the OHPM, we first consider a certain working regime characterized by the following parameters:

$$A = 1\,,\ m = 2\,,\ c = 0.8\,,\ k = 100\,,\ \lambda = 0.03\,,\quad \omega_1 = 1\,,\ \omega_2 = 3\,,\ f = 2\,,\ \Omega = 7.068238819\,. \tag{45}$$

In these conditions, following the procedure based on a collocation-type approach and starting from (32), we obtain the optimal values of the convergence-control parameters:

$$C_1 = 1.76194\,,\ C_2 = -0.249164\,,\quad C_3 = -0.335675\,,\ C_4 = 0.0316235\,. \tag{46}$$

Figure 1 shows the comparison between the numerical solution obtained using a fourth-order Runge–Kutta method and the approximate solution given by (32) with the convergence–control parameters given by (46). It can be observed that the displacement curve obtained for this non-conservative oscillatory system through OHPM is quasi-identical to that obtained via numerical simulation, which proves the validity of the proposed procedure for non-conservative oscillators.

An error analysis is graphically performed in Figure 2, plotting the absolute error $\mathrm{Er}(t) = u_{\mathrm{RK}}(t) - u_{\mathrm{app}}(t)$, where $u_{\mathrm{RK}}$is the numerical solution of (1) obtained using a fourth-order Runge–Kutta algorithm and u_{app} is the approximate solution given by (32) and (46).

However, as we mentioned before, this solution is valid only for transitory behaviour of this oscillator. In order to obtain a uniformly valid solution for large time, based on (39) and the same procedure for obtaining the convergence-control parameters, we get

$$C_1 = -0.196884\,,\ C_2 = -0.00791264\,,\quad C_3 = 0.584621\,,\ C_4 = 0.0270292\,,\ C_5 = 1.17235\,. \tag{47}$$

The comparison between the numerical and the approximate solution (39) and (47) is given in Fig-

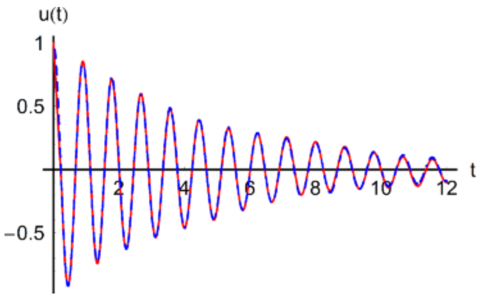

Fig. 1 (colour online). Comparison between the presented solution (32) with (46) and the numerical solution of (1) for $m = 2$, $c = 0.8$, $k = 100$, $\lambda = 0.03$, $\omega_1 = 1$, $\omega_2 = 3$, $f = 2$, —— numerical simulation, - - - - approximate solution.

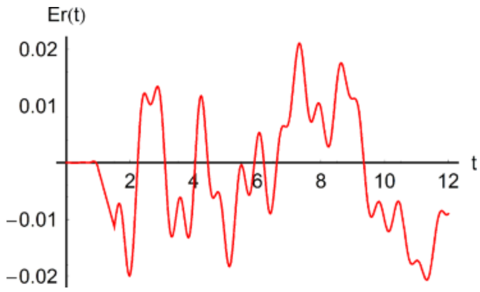

Fig. 2 (colour online). Absolute error between the numerical solution and the approximate solution (32) with (46).

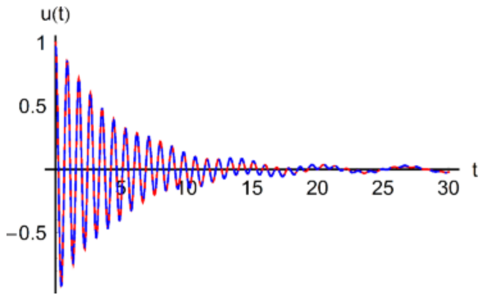

Fig. 3 (colour online). Comparison between the presented solution (39) with (47) and the numerical solution of (1) for $A = 1$, $m = 2$, $c = 0.8$, $k = 100$, $\lambda = 0.03$, $\omega_1 = 1$, $\omega_2 = 3$, $f = 2$, —— numerical simulation, - - - - approximate solution.

ure 3 while the error analysis is graphically performed in Figure 4. One can observe a significant improvement of the error compared to the previous solution, which demonstrates that increasing the number of convergence-control parameters in the auxiliary functions lead to an improved accuracy of the obtained results. Moreover, this solution is uniformly valid for large t, even for $t \longrightarrow \infty$.

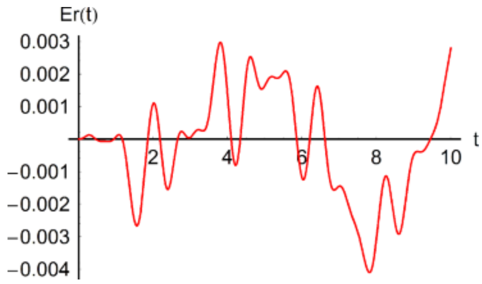

Fig. 4 (colour online). Absolute error between the numerical solution and approximate solution (39) with (47).

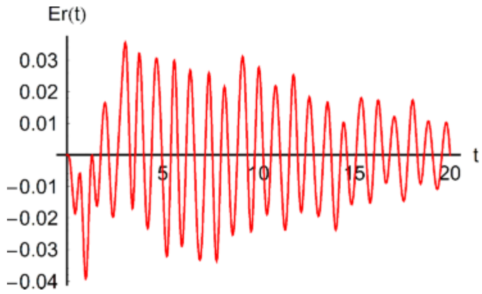

Fig. 5 (colour online). Absolute error between the numerical solution and OHPM solution (39) with (48) for $\lambda = 0.2$, —— numerical simulation, - - - - approximate solution.

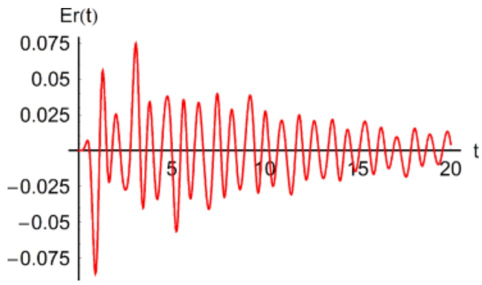

Fig. 6 (colour online). Absolute error between the numerical solution and HPM solution (43) for $\lambda = 0.2$, —— numerical simulation, - - - - approximate solution.

It can be seen that the behaviour of the considered oscillator is very sensitive to the parameter λ. In what follows, we provide a comparison between the first-order approximate solutions obtained through OHPM using five convergence-control parameters and the HPM solutions for various values of the parameter λ.

For $\lambda = 0.2$ and keeping the other initial data for (1), using the procedure described above, we obtain through OHPM the following set of convergence-

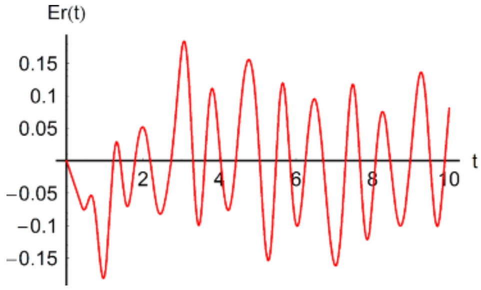

Fig. 7 (colour online). Absolute error between the numerical solution and OHPM solution (39) with (49) for $\lambda = 0.4$, —— numerical simulation, - - - - approximate solution.

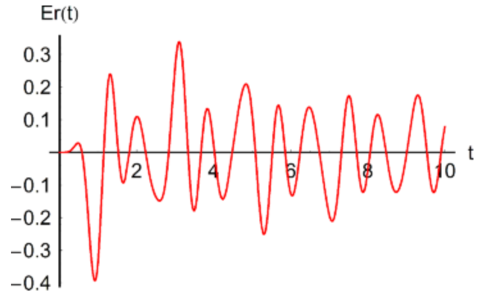

Fig. 8 (colour online). Absolute error between the numerical solution and HPM solution (43) for $\lambda = 0.4$, —— numerical simulation, - - - - approximate solution.

control parameters:

$$C_1 = 0.122285\,, \quad C_2 = -0.00305739\,, \\ C_3 = 0.46671\,, \quad C_4 = 0.0863294\,, \quad C_5 = 1.16127\,. \tag{48}$$

As a last example, considering $\lambda = 0.4$, the obtained convergence-control parameters are

$$C_1 = 0.437039\,, \quad C_2 = -0.0193715\,, \\ C_3 = 0.308276\,, \quad C_4 = 0.159496\,, \quad C_5 = 1.50562\,. \tag{49}$$

A comparison between the absolute errors obtained for the above first-order approximate solutions given by OHPM and HPM is presented in Figures 5–8 for different values of λ. One can notice that increasing the value of the parameter λ, the error obtained through HPM increases compared to the error obtained through OHPM. OHPM provides low error due to the convergence-control procedure. Moreover, if a better error is needed, OHPM allows increasing the number of convergence-control parameters in the auxiliary functions, which leads to a significant improvement of the error, as shown before.

5. Conclusions

In this paper, a new analytical technique, called the optimal homotopy perturbation method, is applied to a non-conservative dynamical system of a rotating electrical machine. Our construction of the new homotopy is different from that employed in traditional HPM, especially referring to the auxiliary functions $K_{11}(t, C_k)$ and $K_{12}(t, C_k)$ and also to the presence of some convergence-control parameters $C_1, C_2, \ldots$. These parameters ensure a rapid convergence of the solution to the exact one when they are optimally determined.

Using the proposed procedure for the investigated oscillator, a solution corresponding to the transitory behaviour has been obtained as well as a uniformly valid solution, even for $t \to \infty$. It was proved that the more convergence-control parameters are used in the auxiliary functions, the more accurate will be the obtained results. A slightly increased number of convergence-control parameters in the auxiliary functions leads to a significant improvement of the error of the approximate results.

It was observed that the behaviour of the considered oscillator is very sensitive to the parameter λ. Therefore comparisons performed between OHPM and HPM results for different values of the parameter λ reveal that both methods lead to accurate results, but the results obtained through OHPM are more accurate than the results obtained through HPM for the same order of approximation, for large parameters, due to the convergence-control procedure employed by OHPM.

The main feature of the OHPM is that it provides a simple and rigorous way to control the solution through several parameters C_k which are optimally determined.

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