Non-Newtonian Micropolar Fluid Squeeze Film Between Conical Plates

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By applying the micropolar fluid model of Eringen (J. Math. Mech. **16**, 1 (1966) and Int. J. Mech. Sci. **31**, 605 (1993)), the squeeze film lubrication problems between conical plates are extended in the present paper. A non-Newtonian modified Reynolds equation is derived and applied to obtain the solution of squeeze film characteristics. Comparing with the traditional Newtonian case, the non-Newtonian effects of micropolar fluids are found to enhance the load capacity and lengthen the approaching time of conical plates. Some numerical results are also provided in tables for engineer applications.

Key words: Micropolar Fluids; Conical Plates; Squeeze Films; Load-Carrying Capacity; Approaching Time.

1. Introduction

The study of squeeze film mechanisms shows great importance in practical applications, such as the human joints, hydraulic dampers, engine components, machine tools, etc. Traditionally, the use of a Newtonian lubricant is applied to analyze the load capacity and the approaching time of squeeze films. Typical studies can be observed in the squeezing rectangular plates and the parallel disks by Moore [1], the squeezing journal bearings by Hays [2], and the squeeze-film conical bearings by Prakash and Vij [3] and Khonsaari and Booser [4]. Owing to the development and the severe operating conditions of modern machine systems, increasing attention has been emphasized on the use of non-Newtonian lubricants. According to the experimental contributions of Oliver and Shahidullah [5] and Oliver [6], the addition of long-chained polymers in the oils provides decreased friction coefficient and increased load capacity for squeeze film plates and journal bearing systems. From the experimental researches of Spikes [7] and Scott and Suntiwattana [8], a lubricant mixed with a small amount of additives shows beneficial effects on the friction characteristics and can reduce the wear of friction surface. In order to describe the flow behaviour of such kinds of fluids containing microstructures, a microcontinuum theory of micropolar fluids has been developed by Eringen [9, 10]. This non-Newtonian micropolar fluid model can be applied to study the flows of liquid crystals, human bloods, polymer-thickened lubricants, and fluids with additives. By applying the micropolar fluid model, several contributions have been presented, such as the parallel plates with reference to human joints by Nigam et al. [11], the journal bearings by Naduvinamani and Huggi [12], the sphere-plate mechanisms by Al-Fadhalah and Elsharkawy [13] and Lin et al. [14], and the hemispherical bearings by Sinha and Singh [15]. According to their results, the non-Newtonian effects of micropolar fluids show significant influences on the squeeze film performances. Since the studies of squeeze film characteristics between conical plates are also important in engineering applications [3, 4], a further investigation by using the micropolar fluids as lubricants is of interest.

In the present study, the squeeze film problems between conical plates [4] are extended by applying Eringen's micropolar fluid model [9, 10]. A modified Reynolds equation is derived by using the linear momentum, angular momentum, and continuity equation. Closed-form expressions for the load capacity and the approaching time are also obtained. Comparing with the traditional Newtonian case, the non-Newtonian effects of micropolar fluids upon the conical-plate characteristics are displayed through the variation of non-Newtonian parameters and conical angles. Tables with numerical results are also included for bearing designing and selection.

2. Analysis

Figure 1 shows the squeeze film configuration of a conical-plate bearing lubricated with an incompressible micropolar fluid. The cone with radius R and angle 2θ is approaching the bearing housing. The film thickness along the vertical direction is h. Based upon the micropolar fluid model of Eringen [9, 10], the two-dimensional field equations neglecting the body forces and the body couples can be expressed in polar coordinates as

$$\text{Continuity equation: } \frac{1}{x}\frac{\partial}{\partial x}(xu)+\frac{\partial v}{\partial y}=0\,, \tag{1}$$

$$\text{Linear momentum: } \frac{1}{2}(2\mu+k)\frac{\partial^2 u}{\partial y^2}+k\frac{\partial s}{\partial y}=\frac{\mathrm{d}p}{\mathrm{d}x}\,, \tag{2}$$

$$\text{Angular momentum: } \gamma\frac{\partial^2 s}{\partial y^2}-k\frac{\partial u}{\partial y}-2ks=0\,. \tag{3}$$

In these equations, p denotes the film pressure, u and v describes the velocity components in the x- and y-directions, respectively, s represents the microrotational velocity component, μ is the traditional viscosity coefficient, γ is the additional viscosity coefficient, and k is the additional spin viscosity coefficient of micropolar fluids. When $k = 0$, the non-Newtonian influences of micropolar fluids disappear, and the Navier–Stokes equation is recovered from the momentum equations (2) and (3).

The boundary conditions for the fluid at the cone and housing surfaces are:

$$\text{at } y=0:\;\; u=0\,,\;\; s=0\,,\;\; v=0\,, \tag{4}$$

$$\text{at } y=h\sin\theta:\;\; u=0\,,\;\; s=0\,,\;\; v=\sin\theta\frac{\mathrm{d}h}{\mathrm{d}t}\,. \tag{5}$$

The simultaneous differential (2) and (3) are solved by using the boundary conditions for u and s. As a result, one can obtain

$$u=\frac{1}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot\left\{\,y^2-yh\sin\theta-\frac{C^2}{m}h\sin\theta\left[\frac{\cosh(my)+\cosh[m(h\sin\theta-y)]-\cosh(mh\sin\theta)-1}{\sinh(mh\sin\theta)}\right]\,\right\}, \tag{6}$$

$$s=\frac{h\sin\theta}{2\mu}\frac{\mathrm{d}p}{\mathrm{d}x}\cdot\left\{\,\frac{\sinh(my)+\sinh(mh\sin\theta)-\sinh[m(h\sin\theta-y)])}{2\sinh(mh\sin\theta)}-\frac{y}{h\sin\theta}\,\right\}, \tag{7}$$

where $m=C/l$ and

$$l=(\gamma/4\mu)^{1/2}\,, \tag{8}$$

$$C=[k/(2\mu+k)]^{1/2}\,. \tag{9}$$

Substituting the expression of u into the integrated continuity equation (6) across the film thickness and applying the boundary conditions of u and v, one gets

$$\frac{1}{x}\cdot\int_{y=0}^{y=h\sin\theta}\frac{\partial}{\partial x}(xu)\,\mathrm{d}y=-\int_{y=0}^{y=h\sin\theta}\frac{\partial v}{\partial y}\,\mathrm{d}y\,. \tag{10}$$

After performing the integration, one can derive the modified Reynolds equation for the squeeze-film conical plates lubricated with a non-Newtonian micropolar-fluid:

$$\frac{1}{x}\cdot\frac{\mathrm{d}}{\mathrm{d}x}\left(x\cdot\frac{\mathrm{d}p}{\mathrm{d}x}\right)=12\mu\frac{\sin\theta}{\varphi(h,l,m,\theta)}\cdot\frac{\mathrm{d}h}{\mathrm{d}t}\,, \tag{11}$$

$$\begin{aligned}\varphi&=h^3\sin^3\theta+12l^2h\sin\theta\\&\quad-6l^2mh^2\sin^2\theta\cdot\coth(0.5mh\sin\theta)\,.\end{aligned} \tag{12}$$

The boundary conditions for the film pressure are:

$$\text{at } x=0:\;\; \frac{\mathrm{d}p}{\mathrm{d}x}=0\,, \tag{13}$$

$$\text{at } x=R\csc\theta:\;\; p=0\,. \tag{14}$$

Integrating the modified Reynolds equation and using the boundary conditions yield the expression of the film pressure,

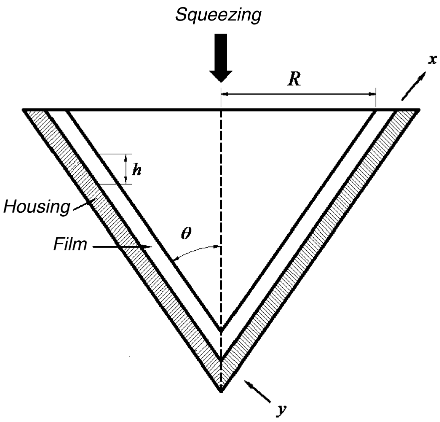

Fig. 1. Squeeze film configuration of conical plates lubricated with a non-Newtonian micropolar fluid.

$$p = \frac{3\mu v_s}{\varphi(h,l,m,\theta)} \cdot (R^2 \csc^2\theta - x^2), \tag{15}$$

where $v_s = -\sin\theta \cdot (\mathrm{d}h/\mathrm{d}t)$. Expressed in a non-dimensional form, one can obtain

$$p^* = \frac{3\csc^2\theta}{\varphi^*(h^*,I,C,\theta)} \cdot (1 - x^{*2}), \tag{16}$$

where

$$\varphi^* = h^{*^3}\sin^3\theta + 12I^2h^*\sin\theta - 6ICh^{*2}\sin^2\theta \cdot \coth(0.5(C/I)h^*\sin\theta). \tag{17}$$

In these equations, the following non-dimensional variables and parameters have been introduced:

$$x^* = \frac{x}{R\csc\theta}, \quad h^* = \frac{h}{h_0}, \quad p^* = \frac{ph_0^3}{\mu R^2 v_s}, \quad \varphi^* = \frac{\varphi}{h_0^3}, \quad I = \frac{l}{h_0}. \tag{18}$$

Integrating the film pressure over the film region gives the load-carrying capacity,

$$W = 2\pi \cdot \int_{x=0}^{R\csc\theta} p \cdot x\,\mathrm{d}x. \tag{19}$$

Performing the integration and arranging the results, the load capacity in a non-dimensional form can be obtained:

$$W^* = \frac{Wh_0^3}{\mu\pi R^4 v_s} = \frac{3\csc^4\theta}{2\varphi^*(h^*,I,C,\theta)}. \tag{20}$$

Now, the non-dimensional time is introduced as follows:

$$T^* = \frac{Wh_0^2}{\mu\pi R^4}t. \tag{21}$$

Substituting into (20), one can obtain a first-order differential equation governing the film thickness:

$$\frac{\mathrm{d}h^*}{\mathrm{d}T^*} = -\frac{2\varphi^*(h^*,I,C,\theta)}{3\csc^3\theta}. \tag{22}$$

The initial condition for the film thickness is

$$h^*(T^* = 0) = 1. \tag{23}$$

After solving the differential equation, on can obtain the approaching time required for the cone to reduce the film thickness from the initial value $h^* = 1$ to a given thickness h^*,

$$T^* = \frac{3\csc^3\theta}{2} \cdot \int_{h^*}^{h^*=1} \frac{\mathrm{d}h^*}{\varphi^*(h^*,I,C,\theta)}. \tag{24}$$

The time of approach can be numerically obtained by the method of Gaussian quadrature.

3. Results and Discussion

On the ground of the above analysis, the conical-plate characteristics are influenced by the half cone angle θ and the two non-Newtonian parameters including the interacting parameter $I = l/h_0$, defined in (18), and the coupling parameter $C = [k/(2\mu + k)]^{1/2}$, defined in (9). When the characteristic material length (defined in (8)) $l = 0$ or the additional spin viscosity coefficient $k = 0$, the non-Newtonian effects of micropolar fluids vanish, and the traditional Newtonian conical-plate problem is recovered.

Figure 2 shows the load capacity W^* varying with the film height h^* for different I and C under $\theta = 45°$. Comparing with the Newtonian-lubricant situation, the non-Newtonian effects of micropolar fluids ($I = 0.1$, $C = 0.2$) provide a higher load-carrying capacity. Increasing values of the coupling parameter ($I = 0.1$, $C = 0.3$; $C = 0.4$; $C = 0.5$) result in a further increment of the load capacity. When the value of the interacting parameter increases up to $I = 0.5$, the non-Newtonian effects of micropolar fluids on the load capacity are more intensified. Figure 3 presents the film

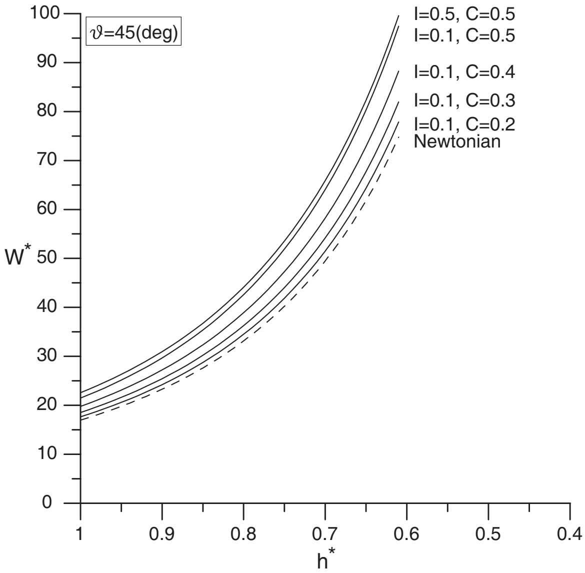

Fig. 2. Load capacity W^* varying with the film height h^* for different I and C.

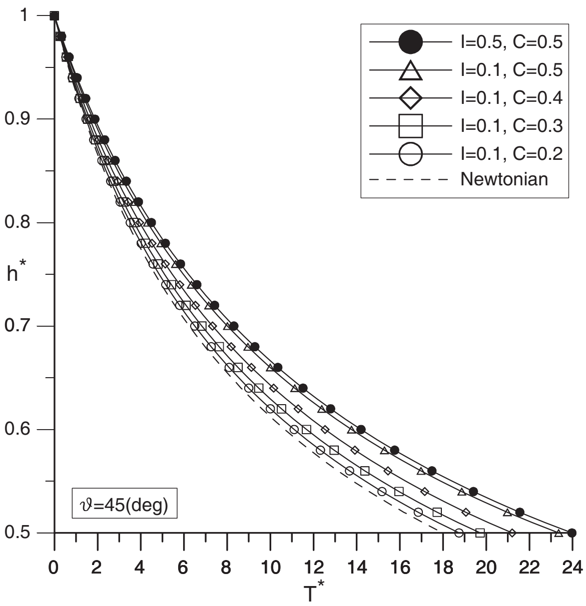

Fig. 3. Film height h^* varying with the approaching time T^* for different I and C.

height h^* varying with the approaching time T^* for different I and C under $\theta = 45°$. The non-Newtonian influences ($I = 0.1$, $C = 0.2$) are observed to yield a longer approaching time as compared to the traditional Newtonian-lubricant case. Increasing values

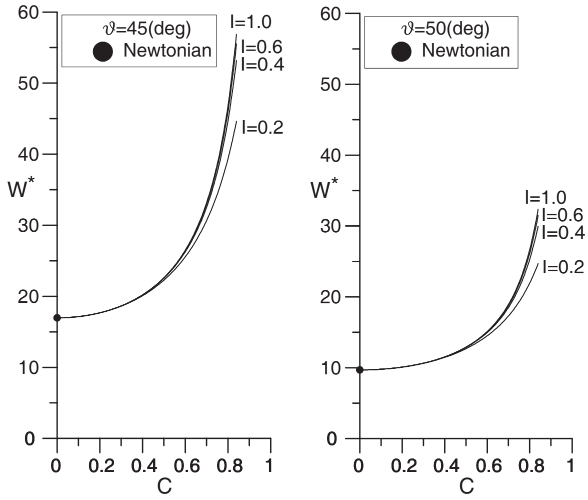

Fig. 4. Load capacity W^* varying with the coupling number C for different I under $h^* = 1$.

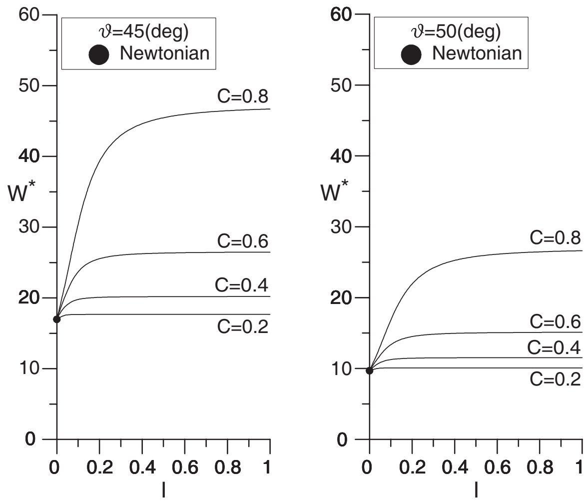

Fig. 5. Load capacity W^* varying with the interacting number I for different C under $h^* = 1$.

of the non-Newtonian parameters ($I = 0.1$, $C = 0.3$; $C = 0.4$; $C = 0.5$; $I = 0.5$, $C = 0.5$) lengthens more the values of T^*.

Figure 4 displays the load capacity W^* varying with the coupling number C for different I under $h^* = 1$. It is observed that for fixed I the effects of the coupling parameter about $C \leq 0.2$ on the value of W^* are small. However, for a coupling parameter $C > 0.2$, the increment of the load due to the micropolar fluids are enlarged especially for larger values of C and I. Figure 5 presents the load capacity W^* varying with the inter-

Table 1. Load capacity W^* for conical plates with $\theta = 55°$ under $h^* = 1$.

			Newtonian case		
$I = 0$ or $C = 0$				6.061	
			Non-Newtonian load capacity		
	$I = 0.1$	$I = 0.2$	$I = 0.3$	$I = 0.4$	$I = 0.5$
$C = 0.1$	6.121	6.122	6.122	6.122	6.122
$C = 0.2$	6.302	6.311	6.312	6.313	6.313
$C = 0.3$	6.603	6.644	6.653	6.656	6.658
$C = 0.4$	7.024	7.158	7.189	7.200	7.206
$C = 0.5$	7.574	7.914	8.002	8.036	8.052

Table 2. Load capacity W^* for conical plates with $\theta = 60°$ under $h^* = 1$.

			Newtonian case		
$I = 0$ or $C = 0$				4.106	
			Non-Newtonian load capacity		
	$I = 0.1$	$I = 0.2$	$I = 0.3$	$I = 0.4$	$I = 0.5$
$C = 0.1$	4.132	4.132	4.132	4.132	4.132
$C = 0.2$	4.268	4.274	4.276	4.276	4.276
$C = 0.3$	4.469	4.500	4.506	4.509	4.510
$C = 0.4$	4.747	4.845	4.868	4.876	4.880
$C = 0.5$	5.104	5.349	5.414	5.440	5.452

acting number I for different C under $h^* = 1$. The effects of the variation of I on the load is slight for small values of C (for example, $C = 0.2$). When the values of the non-Newtonian parameter I and C tend to be large, the non-Newtonian micropolar fluids signify apparent influences on the load capacity. It is also observed that decreasing the half cone angles (from $\theta = 50°$ to $\theta = 45°$) increases the non-Newtonian effects of micropolar fluids upon the cone-plate characteristics.

It is useful to provide numerical results for engineering applications. Tables 1 and 2 present the load capacity W^* for non-Newtonian micropolar-fluid conical plates with half cone angles $\theta = 55°$ and $\theta = 60°$. On the whole, the conical plates lubricated with micropolar fluids provide improved characteristics and lengthen the operating life of squeeze films.

4. Conclusions

By applying the micropolar fluid model of Eringen [9, 10], the squeeze film lubrication problems between conical plates [4] have been extended in the present paper. From the above analysis and results, conclusions are drawn as follows.

A non-Newtonian modified Reynolds equation has been derived by using the linear momentum, angular momentum, and continuity equation. Comparing with the traditional Newtonian-lubricant case, the non-Newtonian influences of micropolar fluids provide higher values of the load capacity and lengthen the time of approach. Numerical results are also included for engineering applications. On the whole, the conical plates lubricated with micropolar fluids provide improved bearing characteristics and therefore lengthen the operating life of squeeze films.

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