Analytical Solutions of the Slip Magnetohydrodynamic Viscous Flow over a Stretching Sheet by Using the Laplace–Adomian Decomposition Method

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In this research, the Laplace–Adomian decomposition method (LADM) is applied for the analytical and numerical treatment of the nonlinear differential equation that describes a magnetohydrodynamic (MHD) flow under slip condition over a permeable stretching surface. The technique is well applied to approximate the similarity solutions of the problem for some typical values of model parameters. The obtained series solutions by the LADM are combined with the Padé approximation to improve the accuracy and enlarge the convergence domain of the obtained results. Through tables and figures, the efficiency of the presented method is illustrated.

Key words: Laplace Adomian Decomposition Method; Padé Approximation; Navier–Stokes Equations; Semi-Infinite Interval; Magnetohydrodynamic Flow.

1. Introduction

The viscous flow induced by a stretching boundary is important in extrusion processes. Many of the boundary layer flows on a stretching surface or the stretching boundary problems have been investigated in literature [1 – 4]. In the recent years, the magnetohydrodynamic (MHD) flow over a stretching sheet in micro-electro-mechanical systems (micro-scale fluid dynamics) received much attention in research. Although, for MHD flow in the micro-scale dimensions, the fluid flow behaviour belongs to the slip flow regime and greatly differs from the classical flow [5], the fluid behaviour still obeys the Navier–Stokes equations, but with slip velocity or temperature boundary conditions [6]. The slip flows have been studied for both impermeable surface and permeable surface in the literature [6 – 8]. Among these papers, Fang et al. in [6] and authors in [7, 8] gave the rare closed-form solutions for the slip flow over a permeable and an impermeable stretching surface, respectively. In investigations of boundary layer problems usually the governing system of the Navier–Stokes equations are transformed into a nonlinear ordinary boundary value problem over a semi-infinite interval by a suitable variable transformation. Then solving these types of equations is a very important subject on which many authors recently focus [9, 10]. The development of new methods is fast-paced, but there are still many issues to be resolved in the thermofluid engineering of small-scale devices, from the fundamental simulation of fluid flow and heat transfer to the optimization of design for fabrication. Boyd [11] replaced the infinite domain with $[-L, L]$ and the semi-infinite interval with $[0, L]$ by choosing a sufficiently large L. Guo [12] converted the model of semi-infinite domains to a problem in a bounded domain, and then solved it by using Jacobi polynomials. In [13 – 16], some spectral methods on unbounded intervals are developed by using mutually orthogonal systems of rational functions. An analytic solution for the time-dependent boundary layer flow over a moving porous surface is derived by using the homotopy analysis method [17].

In recent decade, a new development of the Adomian decomposition method [18, 19], namely the Laplace–Adomian decomposition method [20 – 22], has been applied for solving many problems. Recently, an elegant combination of the Laplace–Adomian decomposition method and the Padé approximation [23] has been applied for solving some boundary layer problems which involve a boundary condition at infinity [24 – 26]. In this work, we will apply the Laplace–

Adomian decomposition method coupled with Padé approximation to obtain an analytical solution for the slip MHD viscous flow over a stretching sheet [6].

2. Formulation of the Slip MHD Viscous Flow over a Stretching sheet

In this section, we will investigate an important model of nonlinear problem that describes the two-dimensional laminar flow over a continuously stretching sheet in an electrically conducting quiescent fluid. As given in [6], let (u,v) be the velocity components in the (x,y) directions, respectively, and let p be the pressure. Also let $U_w = U_0(x)$ be the sheet stretching velocity and $v_w = v_w(x)$ the wall mass transfer velocity, which will be determined later. The x-axis runs along the shrinking surface in the direction opposite to the sheet motion, and the y-axis is perpendicular to it. Then the Navier–Stokes (NS) equations for the governing equation can be shown as

$$\begin{aligned} &u_x + v_y = 0\,,\\ &uu_x + vu_y = -\frac{p_x}{\rho} + \nu(u_{xx}+u_{yy}) - \frac{\sigma B^2}{\rho}u\,,\\ &uv_x + vv_y = -\frac{p_y}{\rho} + \nu(v_{xx}+v_{yy})\,, \end{aligned}$$

with the following boundary conditions:

$$\begin{aligned} &u(x,0) = U_0(x) + Lu_y\,,\ \ v(x,0) = v_w(x)\,,\\ &u(x,\infty) = 0\,, \end{aligned}$$

where ν is the kinematic viscosity, ρ the fluid density, σ the electrical conductivity of the fluid, and L a proportional constant of the velocity slip. The magnetic field with strength B is applied in the vertical direction, and the induced magnetic field is neglected. This group of NS equations is valid for small magnetic field strength. To simplify the governing equations, we use the similarity variable $\tau = y\sqrt{\frac{U_0}{\nu}}$ and the similarity functions

$$u = U_0 x f'(\tau)\,,\ \ v = -\sqrt{U_0\nu}\,f(\tau)\,.$$

With these considerations in mind, the wall mass transfer velocity becomes $v_w(x) = -\sqrt{U_0\nu}\,f(0)$ and also the governing NS system transforms to the following similarity equation:

$$\frac{\mathrm{d}^3 f}{\mathrm{d}\tau^3} - \left(\frac{\mathrm{d}f}{\mathrm{d}\tau}\right)^2 + f\frac{\mathrm{d}^2 f}{\mathrm{d}\tau^2} - M^2\left(\frac{\mathrm{d}f}{\mathrm{d}\tau}\right) = 0 \tag{1}$$

with boundary conditions

$$\begin{aligned} &f(0) = \lambda\,,\ \ f'(0) = 1 + \gamma f''(0)\,,\\ &\lim_{\tau\to\infty} f'(\tau) = 0\,, \end{aligned} \tag{2}$$

where λ is the wall mass transfer parameter showing the strength of the mass transfer at the sheet, M the magnetic parameter with $M^2 = -\frac{\sigma B^2}{\rho U_0}$, and γ the velocity slip parameter with $\gamma = L\sqrt{\frac{U_0}{\nu}}$. Now our interest is to obtain the similarity solution of (1) with boundary conditions (2). Authors of [27] have presented an analytical solution for this problem with $\gamma = 0$. Recently, Fang et al. in [6] presented a rare closed-form for the similarity solution of this problem. In current work, we will try to obtain an efficient analytical solution for the problem by using the Laplace–Adomian decomposition method coupled with Padé approximation.

3. The Laplace–Adomian Decomposition Method

In this section, we will apply the Laplace–Adomian decomposition method (LADM) to obtain the similarity solution of the nonlinear ordinary differential equation (1) with boundary conditions (2). For this purpose, based on the LADM, we take the Laplace transformation ($\mathcal{L}$) on both sides of (1):

$$\begin{aligned} \mathcal{L}\{f(\tau)\} = \frac{1}{s^3 - M^2 s}\big(&f''(0) + sf'(0) + (s^2 - M^2)f(0)\\ &+\mathcal{L}\{f'(\tau)^2\} - \mathcal{L}\{f(\tau)f''(\tau)\}\big) \end{aligned} \tag{3}$$

with following initial conditions:

$$f(0) = \lambda\,,\ \ f'(0) = 1 + \gamma\alpha\,,\ \ f''(0) = \alpha\,, \tag{4}$$

where α is an unknown constant to be determined. Now, by substituting the conditions (4) into (3), we get

$$\begin{aligned} \mathcal{L}\{f(\tau)\} = \frac{1}{s^3 - M^2 s}\big(&\alpha + s(1+\gamma\alpha) + (s^2 - M^2)\lambda\\ &+\mathcal{L}\{f'(\tau)^2\} - \mathcal{L}\{f(\tau)f''(\tau)\}\big)\,. \end{aligned} \tag{5}$$

Based on the Adomian decomposition method, we represent the solution of (5) as an infinite series given as

$$f(\tau) = \sum_{n=0}^{\infty} f_n(\tau), \tag{6}$$

where the components $f_n(\tau)$ will be determined recurrently. The nonlinear terms $f'(\tau)^2$ and $f(\tau)f''(\tau)$ appearing in the equation can be usually decomposed by an infinite series of the so-called Adomian polynomials [18, 19]:

$$N_1(f) = f'(\tau)^2 = \left(\sum_{n=0}^{\infty} f_n(\tau)\right)'^2 = \sum_{n=0}^{\infty} A_n\,,$$
$$N_2(f) = f(\tau)f''(\tau) = \left(\sum_{n=0}^{\infty} f_n(\tau)\right)\left(\sum_{n=0}^{\infty} f_n(\tau)\right)'' = \sum_{n=0}^{\infty} B_n\,,$$

where the Adomian polynomials A_n and B_n can be computed from

$$A_n = \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\xi^n} N_1\left(\sum_{i=0}^{\infty} \xi^i f_i(\tau)\right)\right]_{\xi=0}\,,$$
$$B_n = \frac{1}{n!}\left[\frac{\mathrm{d}^n}{\mathrm{d}\xi^n} N_2\left(\sum_{i=0}^{\infty} \xi^i f_i(\tau)\right)\right]_{\xi=0}\,.$$

Some of the Adomian polynomials of these nonlinear terms are: $A_0 = f_0'^2$, $A_1 = 2f_0'f_1'$, $A_2 = f_1'^2 + 2f_0'f_2'$, and $B_0 = f_0f_0''$, $B_1 = f_1f_0'' + f_0f_1''$, $B_2 = f_0f_2'' + f_2f_0'' + f_1f_1''$. By substituting the assumed solution and Adomian polynomials into (5), we have

$$\mathcal{L}\left\{\sum_{n=0}^{\infty} f_n(\tau)\right\} = \frac{1}{s^3 - M^2 s}\Bigg(\alpha + s(1+\gamma\alpha) + (s^2 - M^2)\lambda + \mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} - \mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\}\Bigg) = G(s) + \frac{1}{s^3 - M^2 s}\left(\mathcal{L}\left\{\sum_{n=0}^{\infty} A_n\right\} - \mathcal{L}\left\{\sum_{n=0}^{\infty} B_n\right\}\right), \quad (7)$$

where $G(s) = \frac{1}{s^3 - M^2 s}\left(\alpha + s(1+\gamma\alpha) + (s^2 - M^2)\lambda\right)$ represents the term arising from the prescribe initial conditions. Based on the modified Laplace decomposition method [28], the function $G(s)$ can be decomposed into three parts, $G(s) = G_0(s) + G_1(s) + G_2(s)$. Now by comparing both sides of the relation and then applying the inverse Laplace transform $\mathcal{L}^{-1}$, we can obtain following recurrence relations for evaluating f_n, $(n \geq 0)$, as follow:

$$f_0 = \mathcal{L}^{-1}\left\{\frac{1}{s^3 - M^2 s}\left((s^2 - M^2)\lambda\right)\right\},$$
$$f_1 = \mathcal{L}^{-1}\left\{\frac{1}{s^3 - M^2 s}\big(s(1+\gamma\alpha) + \mathcal{L}\{A[0]\} - \mathcal{L}\{B[0]\}\big)\right\}, \quad (8)$$
$$f_2 = \mathcal{L}^{-1}\left\{\frac{1}{s^3 - M^2 s}\big(\alpha + \mathcal{L}\{A[1]\} - \mathcal{L}\{B[1]\}\big)\right\},$$
$$f_i = \mathcal{L}^{-1}\left\{\frac{1}{s^3 - M^2 s}\big(\mathcal{L}\{A[i-1]\} - \mathcal{L}\{B[i-1]\}\big)\right\}, \quad i \geq 3\,.$$

The components of the series solution are

$$f_0 = \lambda\,,$$
$$f_1 = \frac{(1+\gamma\alpha)\sinh(M\tau)}{M}\,,$$
$$f_2 = \frac{-1}{2M^2}\big(2\alpha - 2\alpha\cosh(M\tau) + 2\lambda - 2\lambda\cosh(M\tau) - 2\lambda\cosh(M\tau)\gamma\alpha + \lambda\tau\sinh(M\tau)M + \lambda\tau\sinh(M\tau)M\gamma\alpha + 2\lambda\gamma\alpha\big),$$
$$f_3 = \frac{1}{8M^3}\big(-8M\tau - 8M\tau\gamma^2\alpha^2 - 16M\tau\gamma\alpha - 3M\tau\cosh(M\tau)\lambda^2 - 3M\tau\cosh(M\tau)\lambda^2\gamma\alpha - 4M\tau\cosh(M\tau)\lambda\alpha + \sinh(M\tau)\lambda^2\tau^2M^2 + \sinh(M\tau)\lambda^2\tau^2M^2\gamma\alpha + 3\sinh(M\tau)\lambda^2 + 3\sinh(M\tau)\lambda^2\alpha\gamma + 8\sinh(M\tau)\gamma^2\alpha^2 + 8\sinh(M\tau) + 16\sinh(M\tau)\gamma\alpha + 4\sinh(M\tau)\lambda\alpha\big),$$
$$\vdots$$

Now from (6), we can obtain the approximate analytic solution. The approximate analytic solution at the third iteration scheme is

$$f(\tau) = \frac{1}{8M^3}\big(8\sinh(M\tau)\gamma^2\alpha^2 + 16\sinh(M\tau)\gamma\alpha + 4\sinh(M\tau)\lambda\alpha - 4M\tau\cosh(M\tau)\lambda\alpha + \sinh(M\tau)\lambda^2\tau^2M^2\gamma\alpha - 3M\tau\cosh(M\tau)\lambda^2\gamma\alpha - 8M\lambda\gamma\alpha + 8\sinh(M\tau) - 16M\tau\gamma\alpha - 8M\tau\gamma^2\alpha^2 + 3\sinh(M\tau)\lambda^2\alpha\gamma + \sinh(M\tau)\lambda^2\tau^2M^2 - 3M\tau\cosh(M\tau)\lambda^2 + 8\lambda M^3 + 8M\lambda\cosh(M\tau) + 8M\alpha\cosh(M\tau) - 8M\alpha - 8M\lambda + 8\sinh(M\tau)M^2 + 8M\lambda\cosh(M\tau)\gamma\alpha - 4\lambda\tau\sinh(M\tau)M^2\gamma\alpha + 8\sinh(M\tau)M^2\gamma\alpha - 4\lambda\tau\sinh(M\tau)M^2 - 8M\tau + 3\sinh(M\tau)\lambda^2\big). \quad (9)$$

From (9) it is evident that the obtained analytic solutions through LADM are power series in the independent variable. So, these solutions have not the correct behaviuor at infinity according to the boundary condition $f'(\infty) = 0$, and these solutions cannot be directly applied. Hence, it is essential to combine the series solutions, obtained by the LADM, with the Padé approximants to overcome this problem.

4. The LADM-Padé Approximation

Here we will briefly describe the Laplace–Adomian decomposition Padé approximation. The LADM-Padé approximation for problem (1) is based on the transformation of the power series obtained by the Laplace–Adomian decomposition method (9) into a rational function as

$$[S/N](\tau) = \frac{\sum_{j=0}^{S} a_j \tau^j}{1 + \sum_{j=1}^{N} b_j \tau^j}. \tag{10}$$

In order to have the correct limit at infinity according to the boundary conditions (2), one would expect that $N \geq S$. So the rational function (10) has $S+N+1$ coefficients that we may choose. If $[S/N](\tau)$ is exactly a Padé approximant, then $f(\tau) - [S/N](\tau) = O(\tau^{S+N+1})$. So under such conditions, the coefficients a_j and b_j satisfy

$$\sum_{i=0}^{j} b_i f_{j-i} = a_j, \quad j = 0, \ldots, S, \tag{11}$$

$$\sum_{i=0}^{j} b_i f_{j-i} = 0, \quad j = S+1, \ldots, S+N, \tag{12}$$

where $b_k = 0$ if $k > N$.

From (11) and (12), we can obtain the values of a_i $(0 \leq i \leq S)$ and b_j $(1 \leq j \leq N)$.

5. Numerical Solutions and Results

In this section, we will apply the recursive process (8) obtained from the Laplace–Adomian decomposition method to obtain the similarity solutions of problem (1) for some typical model parameters of λ, γ, and M. From (9) it is clear that the obtained series solutions by using LADM depend on the unknown parameter $\alpha = f''(0)$. So in the first step our purpose is mainly concerned with the physical behaviour of the similarity solution $f(\tau)$ in order to determine the value of the unknown parameter α. For computing this unknown value α, we would apply the boundary condition at infinity. Because the computed approximate solution is a power series, we cannot directly apply the condition at infinity, hence we use the Padé approximant (10) to $f(\tau)$. Though we will construct only diagonal approximants $[M/M]$. Now, by using the presented boundary condition at infinity, we can compute the value α with high accuracy. The values of $f''(0)$ computed by the LADM-Padé approximant for various sets of values of the parameters λ for $M = 2$ and $\gamma = 10$ are shown in Table 1. In Table 2, the obtained values of $f''(0)$ for various sets of values of the parameters γ for $M = 0.5$ and $\lambda = 1$ are presented. Also Table 3 illustrates the obtained values of $f''(0)$ by the presented process for several values of M for $\gamma = 0.5$ and $\lambda = 1$. By these tables, it is evident that the obtained

λ	$\lambda = 5$	$\lambda = 3$	$\lambda = 2$	$\lambda = 1$
[5,5]	−0.098222268	−0.097551378	−0.0970046523	−0.0962547883
[6,6]	−0.098299524	−0.097567286	−0.0970092126	−0.0962556445
[7,7]	−0.098268047	−0.097562943	−0.0970082971	−0.0962555302
[8,8]	−0.098280756	−0.097564115	−0.0970084785	−0.0962555452
[9,9]	−0.098268651	−0.097563519	−0.0970084776	−0.0962555448
[10,10]	−0.098275651	−0.097563798	−0.0970084563	−0.0962555441
exact	−0.098277125	−0.097563868	−0.0970084487	−0.0962555435
λ	$\lambda = 0$	$\lambda = -1$	$\lambda = -2$	$\lambda = -3$
[5,5]	−0.0952646165	−0.0940333114	−0.0926065010	−0.0910540819
[6,6]	−0.0952647182	−0.0940333196	−0.0926065016	−0.0910540820
[7,7]	−0.0952647101	−0.0940333192	−0.0926065015	−0.0910540820
[8,8]	−0.0952647108	−0.0940333192	−0.0926065015	−0.0910540820
[9,9]	−0.0952647107	−0.0940333192	−0.0926065015	−0.0910540820
[10,10]	−0.0952647107	−0.0940333192	−0.0926065015	−0.0910540820
exact	−0.0952647107	−0.0940333192	−0.0926065015	−0.0910540820

Table 1. Numerical results for $f''(0)$ for $M = 2$, $\gamma = 10$, and several values of λ.

Table 2. Numerical results for $f''(0)$ for $M = 0.5$, $\lambda = 1$, and several values of γ.

γ	$\gamma = 0$	$\gamma = 0.5$	$\gamma = 3$	$\gamma = 10$
[6,6]	−1.7531553200	−0.8691110146	−0.2670870355	−0.0926721741
[7,7]	−1.7247751896	−0.8677210541	−0.2667211578	−0.0926198934
[8,8]	−1.7247397703	−0.8676730566	−0.2667862663	−0.0926366158
[9,9]	−1.7247448613	−0.8676739970	−0.2667906771	−0.0926188464
[10,10]	−1.7247448684	−0.8676740828	−0.2667906249	−0.0926339258
exact	−1.7247448714	−0.8676740339	−0.2667906136	−0.0926330284

Table 3. Numerical results for $f''(0)$ for $\lambda = 1$, $\gamma = 0.5$, and several values of M.

M	$M = 0.5$	$M = 1$	$M = 2$	$M = 5$
[4,4]	−0.89300855	−0.96033074	−1.14305001	−1.47049789
[5,5]	−0.85964509	−0.95484683	−1.14207290	−1.47026289
[6,6]	−0.86911101	−0.95619234	−1.14221507	−1.47028666
[7,7]	−0.86772105	−0.95586314	−1.14219477	−1.47028430
[8,8]	−0.86772106	−0.95595323	−1.14219686	−1.47028453
exact	−0.86767403	−0.95593449	−1.14219728	−1.47028451

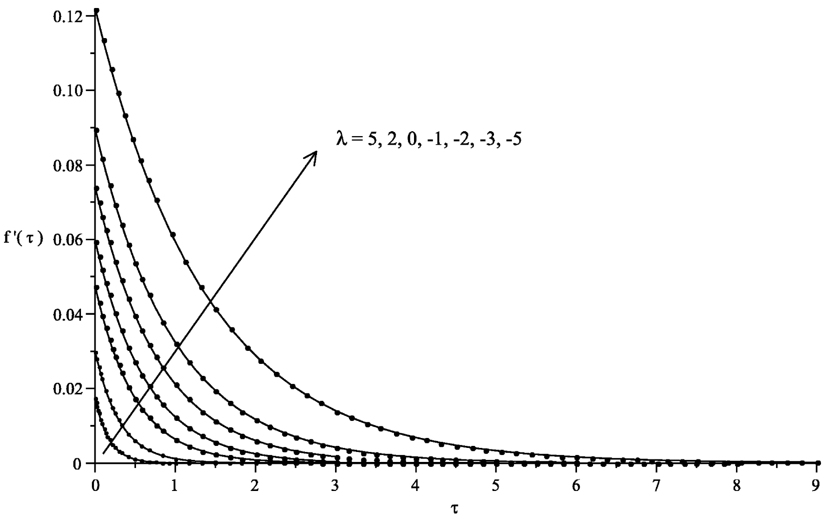

Fig. 1. Comparison between [10/10] LADM-Padé approximate solution (line) with exact solution (point) for $f'(\tau)$ at $M = 2$, $\gamma = 10$, and for several values of λ.

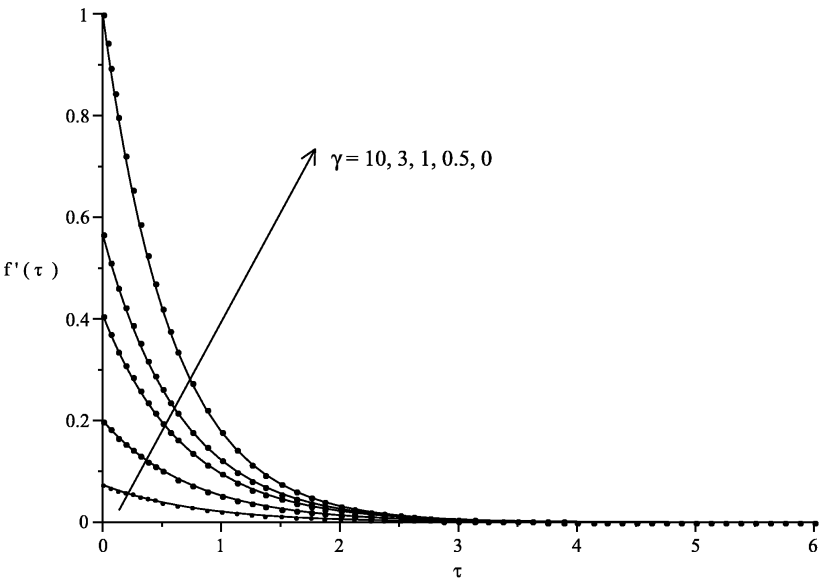

Fig. 2. Comparison between [10/10] LADM-Padé approximate solution (line) with exact solution (point) for $f'(\tau)$ at $M = 0.5$, $\lambda = 1$, and for several values of γ.

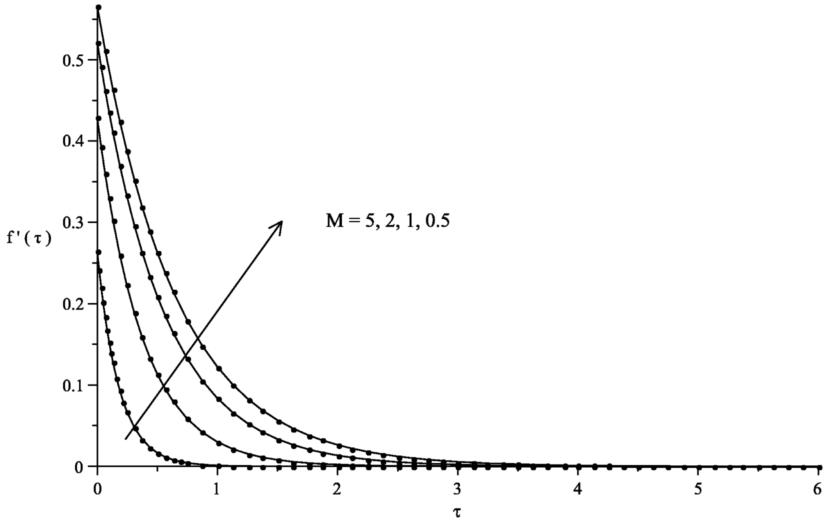

Fig. 3. Comparison between [8/8] LADM-Padé approximate solution (line) with exact solution (point) for $f'(\tau)$ at $\gamma = 0.5$, $\lambda = 1$, and for several values of M.

values for $f''(0)$ by using the LADM-Padé approximant are in excellent agreement with the exact values. In Figure 1, the $[10/10]$ LADM-Padé approximants of $f'(\tau)$ for $M = 2$, $\gamma = 10$, and for various values of λ are plotted and compared with the solution obtained from the closed-form for these cases. Also some of the computed similarity solutions by using the LADM-Padé approximants for $f'(\tau)$ for the viscous values of the model parameters are compared with the exact solutions for these cases, as shown in Figures 2 and 3. From these figures, it is observed that the approximate solutions obtained by the LADM-Padé are in very good agreement with the exact solutions.

6. Conclusions

In the present paper, an efficient technique is used for solving a strong nonlinear ordinary differential equation with the boundary condition at infinity which describes the slip MHD viscous flow over a stretching sheet. To reduce the strong nonlinearity in the governing equation, a Laplace transformation has been applied, and then the Adomian decomposition method coupled with Padé approximant is applied for finding the analytical solutions. The numerical results which are presented through tables and figures imply the effectiveness of the proposed numerical method. It is evident that these types of methods give highly results in very few iterations.

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