A Note on Exact Travelling Wave Solutions for the Klein–Gordon–Zakharov Equations

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In this paper, we investigate the travelling wave solutions for the Klein–Gordon–Zakharov equations by using the modified trigonometric function series method benefited to the ideas of Z. Y. Zhang, Y. X. Li, Z. H. Liu, and X. J. Miao, Commun. Nonlin. Sci. Numer. Simul. **16**, 3097 (2011). Exact travelling wave solutions are obtained.

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1. Introduction

In this paper, we consider the Klein–Gordon–Zakharov equation (KGZE) [1]

$$u_{tt} - u_{xx} + u + \alpha n u = 0, \tag{1}$$

$$n_{tt} - n_{xx} = \beta(|u|^2)_{xx}, \tag{2}$$

where the function $u(x,t)$ denotes the fast time scale component of the electric field raised by electrons, and the function $n(x,t)$ denotes the deviation of the ion density from its equilibrium. Here $u(x,t)$ is a complex function, $n(x,t)$ is a real function, α, β are two nonzero real parameters. This system describes the interaction of the Langmuir wave and the ion acoustic wave in a high frequency plasma. More details are presented in [1] and the references therein.

Recently, applying the trigonometric function series method, Zhang [2] studied the new exact travelling wave solutions of the Klein–Gordon equation

$$u_{tt} - u_{xx} + \alpha u - \beta u^3 = 0. \tag{3}$$

Equation (3) describes the propagation of dislocations within crystals, the Bloch wall motion of magnetic crystals, the propagation of a 'splay wave' along a lied membrane, the unitary theory for elementary particles, the propagation of magnetic flux on a Josephson line, etc. More details are presented in [1]. More recently, some exact solutions for the Zakharov equations are obtained by using different methods [3 – 12]. In [1], using the extended hyperbolic functions method presented in [13], Shang et al. obtained the multiple exact explicit solutions of the KGZEs (1) and (2). These solutions include the solitary wave solutions of bell-type for u and n, the solitary wave solutions of kink-type for u and bell-type for n, the solitary wave solutions of a compound of the bell-type and the kink-type for u and n, the singular travelling wave solutions, the periodic travelling wave solutions of triangle functions type, and solitary wave solutions of rational function type. Especially, Ismail and Biswas [14] investigated the one-soliton solution of the KGZEs (1) with power law nonlinearity by using the solitary wave ansatz method. The solutions are obtained both in $(1+1)$ and $(1+2)$ dimensions. More details are presented in [14]. For the case higher dimensional KGZEs and $\alpha = 1$, $\beta = 1$, by using the methods of dynamical systems, Li [15] considered the existence of exact explicit bounded travelling wave solutions of following equations:

$$\phi_{tt} - \Delta\phi + \phi + \phi\psi u = 0, \quad \psi_{tt} - c^2\Delta\psi = \Delta|\phi|^2, \tag{4}$$

where $\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_n^2}$ is the Laplacian operator, $x \in \mathbb{R}^n$, and c is the propagation speed of a wave. More details are presented in [15].

Quite recently, based on the trigonometric-function series method [2] and the exp-function method [16], Zhang et al. [17] proposed a new method called the modified trigonometric function series method (MTFSM) to construct travelling wave solutions of the perturbed nonlinear Schrödinger equation (NLSE) with Kerr law nonlinearity:

$$\begin{aligned} &\mathrm{i}u_t + u_{xx} + \alpha|u|^2u + \mathrm{i}[\gamma_1 u_{xxx} + \gamma_2|u|^2u_x \\ &+ \gamma_3(|u|^2)_x u] = 0. \end{aligned} \tag{5}$$

However, in our contribution, based on the modified trigonometric function series method (MTFSM), we search travelling wave solutions of KGZEs (1) and (2). More precisely, we combine the trigonometric function series method with the exp-function method. This method is one of the most effective approaches to obtain explicit and exact solutions of nonlinear equations. More details are presented in Section 2.

Remark 1. If we show the Klein–Gordon–Zakharov system in nondimensional variables, it reads

$$\begin{aligned} &c^{-2}u_{tt} - \Delta u + c^2u + nu = 0, \\ &\gamma^{-2}n_{tt} - \Delta n = \Delta|u|^2, \end{aligned} \tag{6}$$

where $u : \mathbb{R}^{1+3} \to \mathbb{R}^3$ is the electric field and $n : \mathbb{R}^{1+3} \to R$ is the density fluctuation of ions, c^2 is the plasma frequency, and γ the ion sound speed, then system (6) describes the interaction between Langmuir waves [18] and ion sound waves in a plasma (see [19] and [20]). Indeed, (1) is the special case of (6). Taking $v = \mathrm{e}^{\mathrm{i}c^2t}u$, system (6) reduces to

$$\begin{aligned} &c^{-2}v_{tt} + 2\mathrm{i}v_t - \Delta v + nv = 0, \\ &\gamma^{-2}n_{tt} - \Delta n = \Delta|v|^2. \end{aligned} \tag{7}$$

Its formal limits $c, \gamma \to \infty$ are given by the nonlinear Schrödinger equation

$$2\mathrm{i}v_t - \Delta v = |v|^2v, \;\; n = -|v|^2. \tag{8}$$

In fact, (8) is the Schrödinger equation with Kerr law nonlinearity. Recently, Biswas and co-worker [21 – 24] studied the solitons of (8).

Remark 2. Obviously, (5) is the perturbation of (8). For the case (5) (in 1D case), it is worth mentioning that Zhang et al. [17, 25 – 27] considered the NLSE (5) with Kerr law nonlinearity and obtained some new exact travelling wave solutions of (5). In [17], by using the modified trigonometric function series method, Zhang et al. studied also some new exact travelling wave solutions of (5). In [25], by using the modified mapping method and the extended mapping method, Zhang et al. derived some new exact solutions of (5), which are the linear combination of two different Jacobi elliptic functions and investigated the solutions in the limit cases. In [26], by using the dynamical system approach, Zhang et al. obtained the travelling wave solutions in terms of bright and dark optical solitons and cnoidal waves. The authors found that (5) has only three types of bounded travelling wave solutions, namely, bell-shaped solitary wave solutions, kink-shaped solitary wave solutions, and Jacobi elliptic function periodic solutions. Moreover, we pointed out the region which these periodic wave solutions lie in. We showed the relation between the bounded travelling wave solution and the energy level h. We observed that these periodic wave solutions tend to the corresponding solitary wave solutions as h increases or decreases. Finally, for some special selections of the energy level h, it is shown that the exact periodic solutions evolute into solitary wave solution. In [27], by using the modified $(\frac{G'}{G})$-expansion method, Miao and Zhang obtained the travelling wave solutions, which are expressed by hyperbolic functions, trigonometric functions, and rational functions. In [28], by using the theory of bifurcations, Zhang et al. investigated the bifurcations and dynamic behaviour of travelling wave solutions to the (5). Under the given parametric conditions, all possible representations of explicit exact solitary wave solutions and periodic wave solutions are obtained.

Remark 3. For system (6), if we take the limit $c \to \infty$, then we get the usual Zakharov system:

$$\begin{aligned} &2\mathrm{i}v_t - \Delta v + nv = 0, \\ &\gamma^{-2}n_{tt} - \Delta n = \Delta|v|^2. \end{aligned} \tag{9}$$

If we take the limit $c \to \infty$, then we get the Klein–Gordon system:

$$c^{-2}u_{tt} - \Delta u + c^2u + |u|^2u = 0. \tag{10}$$

In fact, (3) is the special case of (10). It is classically known that the limit when γ goes to infinity in the Zakharov system (8) leads to the cubic nonlinear Schrödinger equation (9) and that the limit when c goes to infinity in the cubic nonlinear Klein–Gordon sys-

tem (10) also leads to the cubic nonlinear Schrödinger equation.

Remark 4. For (5) without the perturbed term, that is

$$\mathrm{i}u_t + u_{xx} + \alpha|u|^2 u = 0, \tag{11}$$

Ma and Chen [29] studied the exact solutions of (11) with Lie point symmetries and the reflection invariance. Three ansätze of transformations are analyzed and used to construct exact solutions of (11). Various examples of exact solutions with constant, trigonometric function type, exponential function type, and rational function amplitude are given upon careful analysis. A bifurcation phenomenon in (11) is clearly exhibited during the solution process. A general method to generating exact solutions is a multiple exp-function method (see [30]), which is the most general based on Fourier theory. There is also a new solution structure following the linear superposition principle for soliton equations with the Hirota bilinear form (see [31]).

2. New Explicit and Exact Travelling Wave Solutions of (1)

Assume that (1) has travelling wave solutions in the form [1]

$$u(x,t) = \phi(x,t)\exp(\mathrm{i}(kx+\omega t+\xi_0)), \tag{12}$$

where $u(x,t)$ is a real-valued function, k, ω are two real constants to be determined, and ξ_0 is an arbitrary constant. Substituting (12) into (1)–(2) yields

$$\phi_{tt} - \phi_{xx} + (k^2-\omega^2+1)\phi + \alpha n\phi = 0, \tag{13}$$

$$\omega\phi_t - k\phi_x = 0, \tag{14}$$

$$n_{tt} - n_{xx} = \beta(\phi^2)_{xx}. \tag{15}$$

By virtue of (14), we assume

$$\phi(x,t) = \phi(\xi) = \phi(\omega x + kt + \xi_1), \tag{16}$$

where ξ_1 is an arbitrary constant. Substituting (16) into (13), we have

$$n(x,t) = \frac{(\omega-k^2)\phi''(\xi)}{\alpha\phi(\xi)} + \frac{(\omega-k^2-1)}{\alpha}. \tag{17}$$

Hence, we can also assume

$$n(x,t) = \psi(\xi) = \psi(\omega x + kt + \xi_1). \tag{18}$$

Substituting (18) into (15) and integrating the resultant equation twice with respect to ξ, we obtain

$$\psi(\xi) = \frac{\beta\omega^2\phi^2(\xi)}{k^2-\omega^2} + C, \tag{19}$$

where C is an integration constant. It follows from (13) and (19) that

$$\begin{aligned}&\phi''(\xi) + \frac{k^2-\omega^2+1+\alpha C}{k^2-\omega^2}\phi(\xi)\\ &+ \frac{\alpha\beta\omega^2}{(k^2-\omega^2)^2}\phi^3(\xi) = 0.\end{aligned} \tag{20}$$

For simplicity, we assume $A = \frac{k^2-\omega^2+1+\alpha C}{k^2-\omega^2}$, $B = \frac{\alpha\beta\omega^2}{(k^2-\omega^2)^2}$, thus (20) leads to the ordinary differential equation (ODE)

$$\phi''(\xi) + A\phi(\xi) + B\phi^3(\xi) = 0. \tag{21}$$

In what follows, we will discuss the travelling wave solutions of (21).

Based on the trigonometric function series method (see [17]), (21) may have the following solutions

$$\phi(\xi) = \frac{\sum_{j=1}^{j=m} \sin^{j-1}\theta(A_j\sin\theta + A_{m+j}\cos\theta) + A_0}{\sum_{j=1}^{j=m} \sin^{j-1}\theta(B_j\sin\theta + B_{m+j}\cos\theta) + B_0}. \tag{22}$$

where A_j, B_j $(j = 0,1,2,\ldots)$ are unknown constants at this moment, and θ satisfy the equation

$$\frac{\mathrm{d}\theta}{\mathrm{d}\xi} = \sin\theta, \tag{23}$$

while m can be determined by partially balancing the highest degree nonlinear term and the derivative terms of higher order in (21). Here it is determined as $m = 1$.

Hence, the solution takes the following form:

$$\phi(\xi) = \frac{A_1\sin\theta + A_2\cos\theta + A_0}{B_1\sin\theta + B_2\cos\theta + B_0}. \tag{24}$$

It follows from (22) that

$$\phi''(\xi) = \frac{\Phi(\sin\theta,\cos\theta)}{(B_1\sin\theta + B_2\cos\theta + B_0)^3}, \tag{25}$$

where

$$\begin{aligned}
&\Phi(\sin\theta,\cos\theta) = \\
&(A_1B_0 + A_1B_2^2 - A_0B_0B_1 - A_2B_1B_2)\cdot\sin\theta \\
&+ (A_0B_1^2 - A_1B_0B_1 + 2A_0B_2^2 - 2A_2B_0B_2)\sin^2\theta \\
&+ (A_1B_2^2 - A_2B_1B_2 - 2A_1B_0^2 + 2A_0B_0B_1)\sin^3\theta \\
&+ (2A_1B_0B_2 - A_2B_0B_1 - A_0B_1B_2)\sin\theta\cos\theta \\
&+ (A_2B_1^2 - A_1B_1B_2 + 2A_0B_0B_2 - 2A_2B_0^2)\sin^2\theta\cos\theta
\end{aligned}$$

and

$$\phi^3(\xi) = \frac{\Psi(\sin\theta,\cos\theta)}{(B_1\sin\theta + B_2\cos\theta + B_0)^3}, \tag{26}$$

where

$$\begin{aligned}
&\Psi(\sin\theta,\cos\theta) = A_0^3 + 3A_0A_2^2 + 3(A_0^2A_1 + A_1A_2^2)\sin\theta \\
&+ 3(A_0A_1^2 - A_0A_2^2)\sin^2\theta + (A_1^3 - 3A_1A_2^2)\sin^3\theta \\
&+ (A_2^3 + 3A_0^2A_2)\cos\theta + 3(A_1^2A_2 - A_2^3)\sin^2\theta\cos\theta.
\end{aligned}$$

So, substituting (24)–(25) and (26) into (21), it results in an algebraic equation about expansion of various $\sin^j\theta$ $(j = 0,1,2,3)$ and $\sin^j\theta\cos\theta$ $(j = 0,1,2)$ as zero, one can obtain a set of algebraic equation about the expansion coefficients A_j and B_j. Namely,

$$\begin{aligned}
&(A_0B_0^2 + A_0B_2^2 + 2A_2B_0B_2) + B(A_0^3 + 3A_0A_2^2) = 0, \\
&(A_1B_0^2 + A_1B_0^2 - A_0B_0B_1) + A(A_1B_0^2 + A_1B_2^2 \\
&+ 2A_0B_0B_1 + 2A_2B_1B_2) + 3B(A_0^2A_1 + A_1A_2^2) = 0, \\
&(A_2^2B_0^2 + A_2B_2^2 + 2A_0B_0B_2) + B(A_2^3 + 3A_0^2A_2) = 0, \\
&(A_0B_1^2 - A_1B_0B_1 + 2A_0B_0^2 - 2A_2B_0B_2) \\
&+ A(A_0B_1^2 - A_0B_2^2 + 2A_1B_0B_1 - 2A_2B_0B_2) \\
&+ 3B(A_0A_1^2 - A_0A_2^2) = 0, \\
&(A_1B_2^2 - A_2B_1B_2 - 2A_1B_0^2 + 2A_0B_0B_1) \\
&+ A(A_1B_1^2 - A_1B_2^2 - 2A_2B_1B_2) + B(A_1^3 - 3A_1A_2^2) = 0, \\
&(2A_1B_0B_2 - A_2B_0B_1 - A_0B_1B_2) \\
&+ 2A(A_0B_1B_2 + A_1B_0B_2 + A_2B_0B_1) = 0, \\
&(A_2B_1^2 - A_1B_1B_2 + 2A_0B_0B_2 - 2A_2B_0^2) \\
&+ A(A_2B_1^2 - A_2B_2^2 + 2A_1B_1B_2) + B(3A_1^2A_2 - A_2^3) = 0.
\end{aligned} \tag{27}$$

With the aid of Mathematica, from (27) we can get

Case 1. $A_0 = A_2 = B_0 = B_2 = 0$, $A = \frac{1}{2}$;

Case 2. $A_0 = -A_2$, $B_0 = -B_2 = \pm\sqrt{\frac{B}{A}}A_0$, $A_1 = B_1 = 0$;

Case 3. $A_0 = A_2 = B_1 = B_2 = 0$, $A_1 = \pm\sqrt{\frac{-2}{B}}$, $A = -1$;

Case 4. $A_0 = A_2$, $A_1 = B_1$, $B_0 = B_2 = \pm\sqrt{2B}A_0$, $A = \frac{1}{2}$;

Case 5. $A_2 = B_0 = 0$, $B_1 = \pm\sqrt{\frac{B}{A}}\sqrt{A_1^2 - A_0^2}$, $B_2 = \pm\sqrt{\frac{B}{A}}A_0$, $(|A_1| > |A_0|)$;

Case 6. $A_1 = A_2 = B_0 = B_1 = 0$, $B_2 = \pm\sqrt{\frac{B}{2}}A_0$, $A = 2$;

Case 7. $A_0 = B_0 = 0$, $B_1 = \pm\sqrt{2B}A_1$, $B_2 = \pm\sqrt{2B}A_2$, $A = \frac{1}{2}$;

Case 8. $A_2 = B_2 = 0$, $B_0 = \pm\sqrt{\frac{B}{A}}A_0$, $B_1 = \pm\sqrt{\frac{B}{A}}A_1$;

Case 9. $A_0 = B_2 = 0$, $B_1 = \pm\sqrt{2B}\sqrt{A_1^2 + A_2^2}$, $B_0 = \pm\sqrt{2B}A_2$, $A = \frac{1}{2}$;

Case 10. $A_0 = A_1 = B_1 = B_2 = 0$, $B_0 = \pm\sqrt{\frac{B}{2}}A_2$, $A = 2$;

Case 11. $A_1 = B_1 = 0$, $B_0 = \pm\sqrt{\frac{B}{A}}A_0$, $B_2 = \pm\sqrt{\frac{B}{A}}A_2$;

Case 12. $A_2 = -A_0$, $A_1 = B_1 = 0$, $B_2 = -B_0$, $A = -1$;

Case 13. $A_1 = B_1 = 0$, $B_2 = \pm\sqrt{\frac{B}{A}}A_0$, $B_0 = \pm\sqrt{\frac{B}{2}}A_2$, $A = 2$;

Case 14. $A_1 = 0$, $B_0 = \pm\sqrt{2B}A_2$, $B_1 = \pm\sqrt{2B}\cdot\sqrt{A_2^2 - A_0^2}$, $B_2 = \pm\sqrt{2B}A_0$, $A = \frac{1}{2}$, $(|A_2| > |A_0|)$.

In the Cases 1, 2, 4, 7, 8, 11–13, the solution are constants. We find the following types of solutions of (24):
In Case 3:

$$\phi_{1,2}(\xi) = \frac{A_1}{B_0}\sin\theta = \pm\frac{1}{B_0}\sqrt{\frac{2A}{B}}\frac{1}{\cosh(\xi+\eta)}; \tag{28}$$

In Case 5:

$$\phi_{3,4}(\xi) = \frac{A_0 + A_1\sin\theta}{B_1\sin\theta + B_2\cos\theta}$$

$$= \frac{A_0 \pm A_1 \frac{1}{\cosh(\xi+\eta)}}{\pm\sqrt{\frac{B}{A}}\sqrt{A_1^2 - A_0^2}\frac{1}{\cosh(\xi+\eta)} - A_0 \tanh(\xi+\eta)} \quad (29)$$

$(|A_1| > |A_0|)$;

In Case 6:

$$\phi_{5,6}(\xi) = \frac{A_0}{B_2 \cos\theta} = \mp\sqrt{\frac{B}{A}}\frac{A_0^2}{B_2}\frac{1}{\tanh(\xi+\eta)}; \quad (30)$$

In Case 9:

$$\phi_{7,8}(\xi) = \frac{A_1 \sin\theta + A_2 \cos\theta}{B_0 + B_1 \sin\theta} = \frac{\pm A_1 \frac{1}{\cosh(\xi+\eta)} - A_2 \tanh(\xi+\eta)}{\pm\sqrt{\frac{C}{B}}A_0 \mp \frac{1}{\cosh(\xi+\eta)}}; \quad (31)$$

In Case 10:

$$\phi_{9,10}(\xi) = \frac{A_2 \cos\theta}{B_0} = \pm\sqrt{\frac{B}{A}}\tanh(\xi+\eta); \quad (32)$$

In Case 14:

$$\phi_{11,12}(\xi) = \frac{A_0 + A_2 \cos\theta}{B_0 + B_1 \sin\theta + B_2 \cos\theta} = \frac{A_0 - A_2 \tanh(\xi+\eta)}{\pm\sqrt{\frac{C}{B}}A_2 \pm \sqrt{\frac{B}{A}}\sqrt{A_2^2 - A_0^2}\frac{1}{\cosh(\xi+\eta)} - \sqrt{\frac{C}{B}}A_0 \tanh(\xi+\eta)}, \quad (|A_2| > |A_0|). \quad (33)$$

In what follows, $|u|$ is the norm of u. From (28)–(33) and (12), we establish the following travelling solutions of NLSE (1):

$$|u_{1,2}(x,t)| = \left|\frac{1}{B_0}\sqrt{\frac{2A}{B}}\frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)}\right|; \quad (34)$$

$$|u_{3,4}(x,t)| = \left|\frac{A_0 \pm A_1 \frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)}}{\pm\sqrt{\frac{B}{A}}\sqrt{A_1^2 - A_0^2}\frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)} - A_0 \tanh((kx+\omega t+\xi_0)+\eta)}\right|, \quad (|A_1| > |A_0|); \quad (35)$$

$$|u_{5,6}(x,t)| = \left|\sqrt{\frac{B}{A}}\frac{A_0^2}{B_2}\frac{1}{\tanh((kx+\omega t+\xi_0)+\eta)}\right|; \quad (36)$$

$$|u_{7,8}(x,t)| = \left|\frac{\pm A_1 \frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)} - A_2 \tanh(k(x-ct)+\eta)}{\pm\sqrt{\frac{B}{A}}A_0 \mp \frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)}}\right|; \quad (37)$$

$$|u_{9,10}(x,t)| = \left|\sqrt{\frac{B}{A}}\tanh((kx+\omega t+\xi_0)+\eta)\right|; \quad (38)$$

$$|u_{11,12}(x,t)| = \left|\frac{A_0 - A_2 \tanh((kx+\omega t+\xi_0)+\eta)}{\pm\sqrt{\frac{B}{A}}A_2 \pm \sqrt{\frac{B}{A}}\sqrt{A_2^2 - A_0^2}\frac{1}{\cosh((kx+\omega t+\xi_0)+\eta)} - \sqrt{\frac{B}{A}}A_0 \tanh((kx+\omega t+\xi_0)+\eta)}\right|, \quad (|A_2| > |A_0|). \quad (39)$$

3. Conclusion and Discussion

We have presented a direct method for obtaining explicit and exact solutions for the Klein–Gordon–Zakharov equations. These solutions are obtained using a modified trigonometric function series method (MTFSM). More precisely, we combine the trigonometric function series method with the exp-function method. To our knowledge, these results have not been reported in the literature. This method is one of the most effective approaches to obtain explicit and exact solutions of nonlinear equations.

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