Heat Transfer Analysis on the Magnetohydrodynamic Flow of a Non-Newtonian Fluid in the Presence of Thermal Radiation: An Analytic Solution

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In this paper, a two-dimensional, steady magnetohydrodynamic flow and heat transfer analysis of a non-Newtonian fluid in a channel with a constant wall temperature are considered in the presence of thermal radiation. The steady Navier–Stokes equations are reduced to nonlinear ordinary differential equations by using similarity variables. The homotopy perturbation method is used to solve the nonlinear ordinary differential equations. The effects of the pertinent parameters on the velocity and temperature field are discussed.

Key words: Thermal Radiation; Magnetohydrodynamic (MHD) Flow; Homotopy Perturbation Method (HPM).

1. Introduction

The effects of thermal radiation on the flow field in the case of force and natural convection are important in the context of space technology and processes involving high temperatures. In the light of these various applications, England and Emery [1] studied the thermal radiation effect of an optically thin gray gas bounded by a stationary vertical plate. Raptis [2] studied the radiation effect on the flow of a micropolar fluid past a continuously moving plate. Hossain and Takhar [3] analyzed the effect of radiation using the Rosseland diffusion approximation on mixed convection along a vertical plate with uniform free stream velocity and surface temperature. Duwairi and Damseh [4, 5], Duwairi [6], and Damseh et al. [7] studied the effect of radiation and heat transfer in different geometry for various flow conditions.

The flow of an electrically conducting fluid in the presence of a magnetic field is of importance in various areas of technology and engineering such as magnetohydrodynamic (MHD) power generation, MHD flow meters, MHD pumps, etc. It is generally admitted that a number of astronomical bodies (e.g., the Sun, Earth, Jupiter, magnetic stars, Pulsars) possess fluid interiors and (or least surface) magnetic fields. Many researchers [8 – 10] investigated the MHD flow for Newtonian and non-Newtonian fluids.

There has appeared an increasing interest of scientist and engineers in analytical techniques for studying nonlinear problems. Analytical methods have significant advantages over numerical methods in providing analytic, verifiable, rapidly convergent approximations. Therefore, many different new methods recently have been introduced in some ways to eliminate the small parameter such as the homotopy perturbation transform method [11], the Hamiltonian approach [12 – 15], the variational iteration method [16 – 18], the discrete Jacobi sub-equation method [19], the multiple exp-function method [20], and the Laplace decomposition method [21, 22]. One of the semi-exact methods is the homotopy perturbation method (HPM). He [23 – 28] developed and formulated the HPM by merging the standard homotopy and perturbation. He's HPM is proved to be compatible with the versatile nature of the physical problems and has been applied to a wide class of functional equations [29–38].

The objective of the present paper is to investigate the effects of thermal radiation on heat transfer characteristics of a two-dimensional, steady MHD flow of a non-Newtonian fluid in a channel. Governing equations are reduced to nonlinear ordinary differential equations by using similarity variables. Resulting equations have been solved by a well-known homotopy perturbation method. To the author's knowledge, the current paper represents a new approach to the solution of MHD flow.

2. Homotopy Perturbation Method

To illustrate the homotopy perturbation method, let us consider a nonlinear differential equation as in the following form [23–28]:

$$A(u)-f(r)=0,\quad r\in\Omega, \tag{1}$$

subject to boundary conditions

$$B\left(u,\frac{\mathrm{d}u}{\mathrm{d}n}\right)=0,\quad r\in\Gamma, \tag{2}$$

where A is a general differential operator, B a boundary operator, $f(r)$ is a known analytical function, and Γ is the boundary of the domain Ω. A can be split into two parts: a linear part L and a nonlinear part N. Thus, (1) can be rewritten as

$$L(u)+N(U)-f(r)=0. \tag{3}$$

We construct the homotopy $v(r,p):\Omega\times[0,1]\to\mathbb{R}$ which satisfies by using the homotopy technique as below:

$$\begin{aligned}H(v,p)&=(1-p)[L(v)-L(u_0)]\\&+p[A(v)-f(r)]=0\qquad p\in[0,1],\, r\in\Omega.\end{aligned} \tag{4}$$

Equivalently, (4) is written as

$$\begin{aligned}H(v,p)&=L(v)-L(u_0)+pL(u_0)\\&+p[N(v)-f(r)]=0\qquad p\in[0,1],\, r\in\Omega,\end{aligned} \tag{5}$$

where $p\in[0,1]$ is an embedding parameter and u_0 is an initial approximation of (1) which satisfies the boundary conditions. From (4) and (5), one can obtain

$$H(v,0)=L(v)-L(u_0)=0, \tag{6}$$

$$H(v,1)=A(v)-f(r)=0. \tag{7}$$

The changing process of p from 0 to unity is just that of $v(r,p)$ from u_0 to $u(r)$ which is called deformation while $L(v)-L(u_0)$ and $A(v)-f(r)$ are called homotopy. Let us expand the unknown variable v in the perturbation series about the parameter p as

$$v=v_0+pv_1+p^2v_2+\ldots. \tag{8}$$

Taking the limit $p\to 1$ in (8), one can get the solution of (1) as

$$u=\lim_{p\to 1}v=v_0+v_1+v_2+\ldots. \tag{9}$$

When (4) corresponds to (1), then (9) becomes the approximate solution of (1). Some interesting results have been attained using this method.

3. Formulation of the Problem

The present paper considers the two-dimensional steady MHD flow and heat transfer analysis in the presence of thermal radiation of a non-Newtonian fluid in a channel with a constant wall temperature. We introduce the transformations

$$\begin{aligned}&u=bxf'(\eta),\quad v=-abf(\eta),\\&\eta=y/a,\quad \theta=T/T_w,\end{aligned} \tag{10}$$

where a is the half-channel height and b the stretching constant. The coordinate system is located at the center of the channel. The steady Navier–Stokes equations yield a system of nonlinear ordinary differential equations in the form

$$\begin{aligned}&f'''-\mathrm{R}((f')^2-ff'')-\mathrm{M}^2f'+\alpha(2f'f'''-ff^{(iv)}\\&-(f'')^2)=0,\\&f(0)=0,\ \ f(1)=0,\ \ f'(1)=1,\ \ f''(0)=0,\\&\left(\frac{3N+4}{3N}\right)\theta''+\mathrm{PR}f\theta'+\mathrm{PE}(f'')^2+\alpha\mathrm{PE}(f'(f'')^2\\&-ff''f''')=0,\\&\theta'(0)=0,\ \ \theta(1)=1,\end{aligned} \tag{11}$$

where $'$ denotes the differentiation with respect to η, and the dimensionless quantities are defined through

$$\begin{aligned}&R=\frac{ba^2}{\upsilon},\quad M=\sqrt{\frac{\sigma}{\mu}}B_0a,\quad \alpha=\frac{\alpha_1 b}{k},\\&E=\frac{b^2x^2}{T_wc_p},\quad N=\frac{kk^*}{4\Gamma T_0^3},\quad P=\frac{\mu c_p}{k},\end{aligned} \tag{12}$$

in which, R is the Reynolds number, ν is the kinematic viscosity, M is the Hartman number, σ is the electrical conductivity, μ is the viscosity, α is the visco-elastic parameter, k is the thermal conductivity, E is the local Eckert number, c_p is the specific heat of the fluid, N is the thermal radiation parameter, Γ is the Stefan-Boltzmann constant, and P is the Prandtl number.

In view of (4), the homotopy of (11) can be constructed as follows:

$$\begin{aligned}
&(1-p)L_1(f-f_0) \\
&+p\left[\begin{array}{l} f'''-R((f')^2-ff'')-M^2f'+ \\ \alpha(2f'f'''-ff^{(iv)}-(f'')^2) \end{array}\right]=0, \\
&(1-p)\left(\frac{3N+4}{3N}\right)L_2(\theta-\theta_0) \\
&+p\left[\begin{array}{l} \left(\frac{3N+4}{3N}\right)\theta''+PRf\theta'+PE(f'')^2 \\ +\alpha PE(f'(f'')^2-ff''f''') \end{array}\right]=0.
\end{aligned} \tag{13}$$

Assuming $L_1 f=0$ and $L_2\theta=0$, the following expansions for f and θ can be introduced into (13) :

$$\begin{aligned}
f&=f_0+pf_1+p^2f_2+\cdots, \\
\theta&=\theta_0+p\theta_1+p^2\theta_2+\cdots.
\end{aligned} \tag{14}$$

After some simplifications and rearrangements based on the powers of p-terms, following equations can be obtained:

$$\begin{aligned}
&p^{(0)}: L_1f_0=0 \text{ and } L_2\theta_0=0, \\
&f_0(0)=0,\ f_0(1)=0,\ f_0'(1)=1,\ f_0''(0)=0, \\
&\theta_0'(0)=0,\ \theta_0(1)=1,
\end{aligned} \tag{15}$$

where L_1 and L_2 can be defined as

$$L_1=\frac{\partial^4}{\partial\eta^4} \text{ and } L_2=\frac{\partial^2}{\partial\eta^2}. \tag{16}$$

Initial guesses can be obtained by solving (15) as

$$f_0(\eta)=1/2(\eta^3-\eta) \text{ and } \theta_0(\eta)=\eta^2. \tag{17}$$

Higher-order terms can be determined as

$$\begin{aligned}
&p^{(1)}: L_1f_1+f_0'''-\mathrm{R}\big((f_0')^2-f_0f_0''\big)-\mathrm{M}^2f_0' \\
&+\alpha\big(2f_0f_0'''-f_0f_0^{(iv)}-(f_0'')^2\big)=0, \\
&f_1(0)=0,\ f_1(1)=0,\ f_1''(0)=0,\ f_1'(1)=0, \\
&p^{(1)}: \left(\frac{3N+4}{3N}\right)(L_2\theta_1-2)+\mathrm{PR}f_0\theta_0'+\mathrm{PE}\big(f_0''\big)^2 \\
&+\alpha\mathrm{PE}\big(2f_0'\big(f_0''\big)^2-f_0f_0''f_0'''\big)=0, \\
&\theta_1'(0)=0,\ \theta_1(1)=0, \\
&\vdots
\end{aligned} \tag{18}$$

$$\begin{aligned}
&p^{(j)}: L_1f_j-L_1f_{j-1}+f_{j-1}'''-\mathrm{R} \\
&\cdot\left(\sum_{k=0}^{j-1}f_k'f_{j-1-k}'-\sum_{k=0}^{j-1}f_kf_{j-1-k}''\right)-\mathrm{M}^2f_{j-1}'+\alpha \\
&\cdot\left(2\sum_{k=0}^{j-1}f_k'f_{j-1-k}'''-\sum_{k=0}^{j-1}f_kf_{j-1-k}^{iv}-\sum_{k=0}^{j-1}f_k''f_{j-1-k}''\right)=0, \\
&f_j(0)=0,\ f_j'(1)=0,\ f_j(1)=0,\ f_j''(0)=0, \\
&p^{(j)}: \left(\frac{3N+4}{3N}\right)L_2\theta_j-\left(\frac{3N+4}{3N}\right)L_2\theta_{j-1} \\
&+\mathrm{PR}\sum_{k=0}^{j-1}\theta_k'f_{j-1-k}+\mathrm{PE}\sum_{k=0}^{j-1}f_k''f_{j-1-k}''+\alpha\mathrm{PE} \\
&\cdot\left(\sum_{k=0}^{j-1}f_{j-1-k}'\sum_{l=0}^{k}f_{k-l}''f_l''-\sum_{k=0}^{j-1}f_{j-1-k}\sum_{l=0}^{k}f_{k-l}''f_l'''\right)=0, \\
&\theta_j'(0)=0,\ \theta_j(1)=0, \\
&\vdots
\end{aligned} \tag{19}$$

By solving (18), the first-order approximation can be determined by symbolic calculation with software like Mathematica, Maple or Matlab as

$$\begin{aligned}
&f_1(\eta)=-\frac{\eta}{16}-\frac{\mathrm{M}^2\eta}{240}+\frac{17\mathrm{R}\eta}{2688}+\frac{\alpha\eta}{16}+\frac{3\eta^3}{16} \\
&+\frac{\mathrm{M}^2\eta^3}{48}-\frac{11\mathrm{R}\eta^3}{640}-\frac{3\alpha\eta^3}{16}-\frac{\eta^4}{8}-\frac{\mathrm{M}^2\eta^4}{48} \\
&+\frac{\mathrm{R}\eta^4}{96}-\frac{\alpha\eta^4}{8}+\frac{\mathrm{M}^2\eta^6}{240}+\frac{\mathrm{R}\eta^8}{2240}+\frac{1}{2}(\eta^3-\eta), \\
&\theta_1(\eta)=1+\frac{4}{3N}+\frac{3\mathrm{EP}}{4}-\frac{\mathrm{PR}}{20}+\frac{3\mathrm{EP}\alpha}{10}-\frac{4}{3N}\eta^2 \\
&-\frac{3}{4}\mathrm{EP}\eta^4+\frac{1}{12}\mathrm{PR}\eta^4-\frac{1}{30}\mathrm{PR}\eta^6-\frac{3}{10}\mathrm{EP}\alpha\eta^6.
\end{aligned} \tag{20}$$

4. Results and Discussion

Numerical calculations have been carried out for different values of α (second-grade parameter), R (Reynolds number), M (Hartman number), N (thermal radiation parameter), P (Prandtl number), and E (local Eckert number). The effects of these parameters on the flow and heat transfer have been demonstrated. Figures 1 and 2 elucidate the influence of α on the velocity components f and f', respectively. It is noticed that f

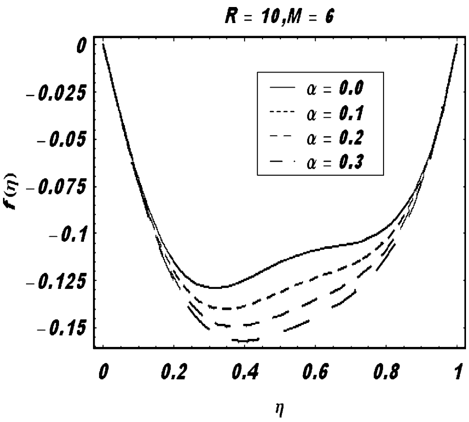

Fig. 1. Variation of f for different values of α.

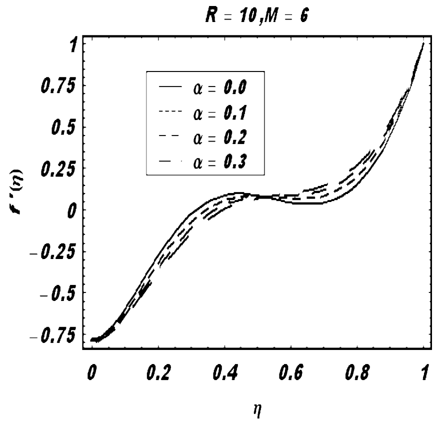

Fig. 2. Variation of f' for different values of α.

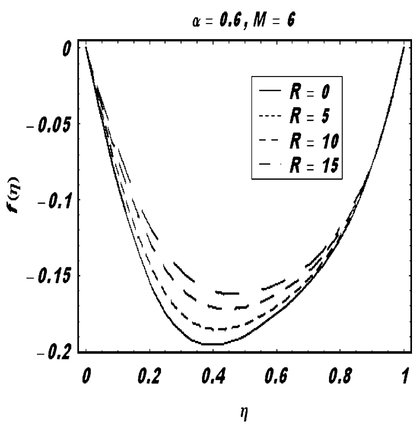

Fig. 3. Variation of f for different values of R.

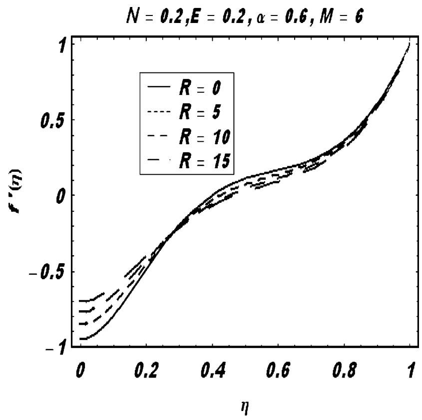

Fig. 4. Variation of f' for different values of R.

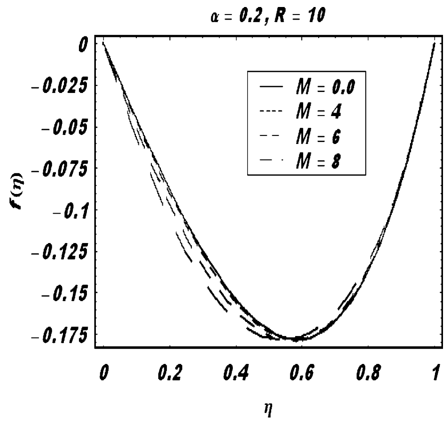

Fig. 5. Variation of f for different values of M.

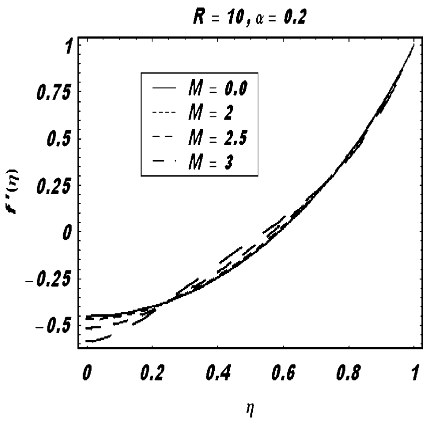

Fig. 6. Variation of f' for different values of M.

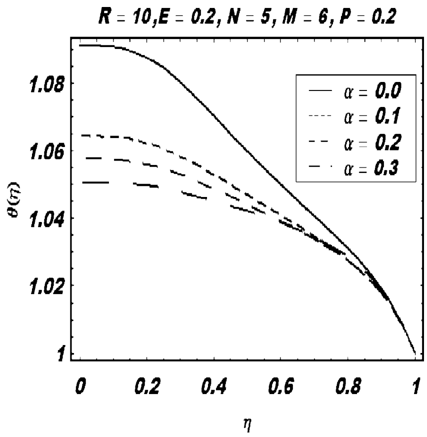

Fig. 7. Variation of θ for different values of α.

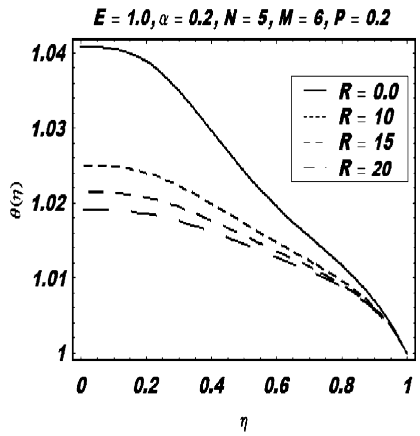

Fig. 8. Variation of θ for different values of R.

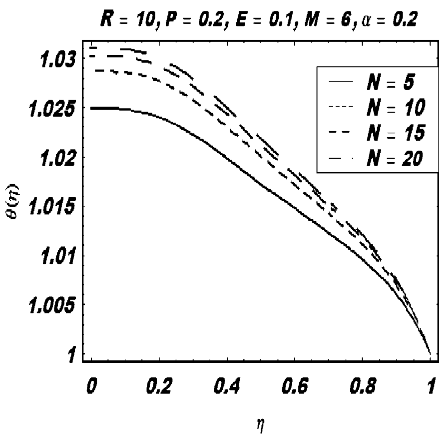

Fig. 9. Variation of θ for different values of N.

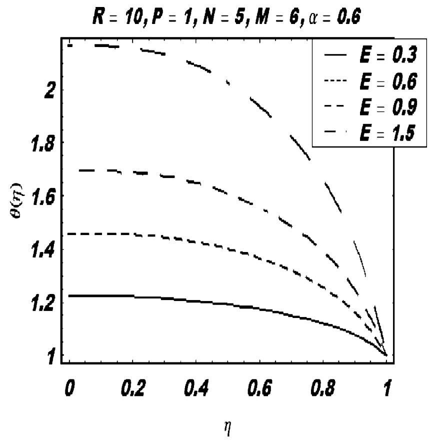

Fig. 10. Variation of θ for different values of E.

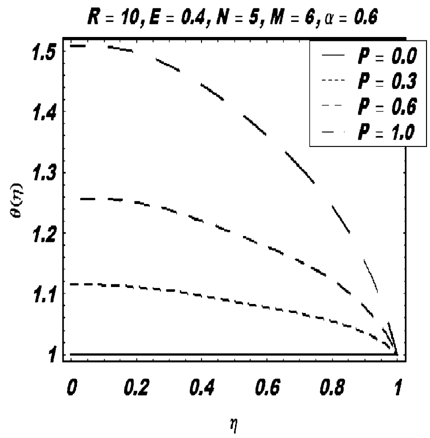

Fig. 11. Variation of θ for different values of P.

decreases with α, however f' first decreases and then increases with α. Figures 3 and 4 show the effect of R on f and f'. It is observed that the effect of R is quite opposite to that of α for both velocity components. Figures 5 and 6 demonstrate the effects of M on the velocity components. As seen from the figures, f decreases with an increase in M up to 0.6. After that point on, the velocity components are independent of M. Figure 6 reveals that f' first decreases with M and then increases. Again after $M = 0.6$, f' is independent of M.

Figures 7 – 11 illustrate the effect of α, R, N, E, and P on the temperature field, respectively. It can be seen from Figures 7 – 9 that the parameters α, R, and N have the same effect on the temperature field. The temperature decreases with increasing α, R, and N, respectively. Similarly, as seen from Figures 10 and 11, E and P show same behaviour. The temperature increases with increasing E and P.

The graphical behaviour of the physical parameters has been calculated for the 10$^{\text{th}}$-order approximation.

5. Conclusion

In this study, the fluid flow and heat transfer characteristics of a two-dimensional, steady MHD flow of a non-Newtonian fluid in a channel with the presence of thermal radiation is studied. Governing equations are transformed into nonlinear ordinary differential equations. The nonlinear equations are solved by HPM, and approximate analytical solutions are determined. The constructed operators are simple and require a lot less computational work. The choice of operators is good enough for the present study, but there are alternative and updated choices of operators [39, 40]. The effects of various key parameters including the visco-elastic parameter α, Hartmann number M, Reynolds number R, thermal radiation parameter N, Prandtl number P, and the local Eckert number E are discussed.

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