Analytical Approach to (2+1)-Dimensional Boussinesq Equation and (3+1)-Dimensional Kadomtsev-Petviashvili Equation

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In this paper, we studied the solitary wave solutions of the (2+1)-dimensional Boussinesq equation $u_{tt} - u_{xx} - u_{yy} - (u^2)_{xx} - u_{xxxx} = 0$ and the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation $u_{xt} - 6{u_x}^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = 0$. By using this method, an explicit numerical solution is calculated in the form of a convergent power series with easily computable components. To illustrate the application of this method numerical results are derived by using the calculated components of the homotopy perturbation series. The numerical solutions are compared with the known analytical solutions. Results derived from our method are shown graphically.

Key words: (2+1)-Dimensional Boussinesq Equation; (3+1)-Dimensional Kadomtsev-Petviashvili Equation; Solitary Wave Solutions; Maple Software Package.

1. Introduction

In this study, we consider the (2+1)-dimensional Boussinesq equation and the (3+1)-dimensional Kadomtsev-Petviashvili (KP) equation:

$$u_{tt} - u_{xx} - u_{yy} - (u^2)_{xx} - u_{xxxx} = 0, \qquad (1)$$

$$u_{xt} - 6{u_x}^2 + 6uu_{xx} - u_{xxxx} - u_{yy} - u_{zz} = 0, \qquad (2)$$

where the initial conditions $u(x,0,t) = f_1(x,t)$, $u_y(x,0,t) = f_2(x,t)$ and $u(x,0,z,t) = g_1(x,z,t)$, $u_y(x,0,z,t) = g_2(x,z,t)$ are given. Nonlinear phenomena play a crucial role in applied mathematics and physics. The studies of the exact solutions for the nonlinear evolution equations have attracted the attention of many mathematicians and physicists [1 – 4]. Senthilvelan [5] studied the travelling wave solutions for the (2+1)-dimensional Boussinesq equation and the (3+1)-dimensional KP equation by the homogeneous balance method and explored certain new solutions of the equations. Recently, El-Sayed and Kaya [6] used the Adomian decomposition method (ADM) for solving this problem.

Finding explicit exact and numerical solutions of nonlinear equations efficiently is of major importance and has widespread applications in numerical analysis and applied mathematics. In this paper, we will represent the homotopy pertubation method (HPM) to find approximate solutions to the (2+1)-dimensional Boussinesq equation and the (3+1)-dimensional KP equation.

The homotopy perturbation method (HPM) was first proposed by the Chinese mathematician Ji-Huan He [7, 8]. Unlike classical techniques, the homotopy perturbation method leads to an analytical approximate and to exact solutions of the nonlinear equations easily and elegantly without transforming the equation or linearizing the problem and with high accuracy, minimal calculation, and avoidance of physically unrealistic assumptions. As a numerical tool, the method provides us with a numerical solution without discretization of the given equation and therefore it is not effected by computation round-off errors and one is not faced with the necessity of large computer memory and time.

The essential idea of this method is to introduce a homotopy parameter, say p, which takes values from 0 to 1. When $p = 0$, the system of equations usually reduces to a sufficiently simplified form, which normally admits a rather simple solution. As p is gradually increased to 1, the system goes through a sequence of 'deformations', the solution for each of which is 'close' to that at the previous stage of 'deformation'. Eventually at $p = 1$, the system takes the original form of the equation and the final stage of 'deformation' gives the desired solution. One of the most remarkable features of the HPM is that usually just a few perturbation terms are sufficient for obtaining a reasonably accurate solution. This technique has been employed to solve a large variety of linear and nonlinear problems [9–24]. The interested reader can see the References [25–28] for last development of HPM.

2. The Homotopy Perturbation Method

Consider the following nonlinear differential equation:

$$A(u)-f(r)=0, \quad r\in\Omega, \tag{3}$$

with boundary conditions

$$B\left(u,\frac{\partial u}{\partial n}\right)=0 \quad r\in\Gamma, \tag{4}$$

where A is a general differential operator, B is a boundary operator, $f(r)$ is a known analytic function, Γ is the boundary of the domain Ω.

The operator A can, generally speaking, be divided into two parts L and N, where L is linear and N is nonlinear, therefore (3) can be written as,

$$L(u)+N(u)-f(r)=0. \tag{5}$$

By using the homotopy technique, one can construct a homotopy $v(r,p):\Omega\times[0,1]\to\Re$ which satisfies

$$H(v,p)=(1-p)[L(v)-L(u_0)]+p[A(v)-f(r)] =0 \tag{6a}$$

or

$$H(v,p)=L(v)-L(u_0)+pL(u_0)+p[N(v)-f(r)] =0, \tag{6b}$$

where $p[0,1]$ is an embedding parameter and u_0 is the initial approximation of (3) which satisfies the boundary conditions. Clearly, we have

$$H(v,0)=L(v)-L(u_0)=0 \tag{7}$$

or

$$H(v,1)=A(v)-f(r)=0. \tag{8}$$

The changing process of p from zero to unity is just that of $v(r,p)$ changing from $u_0(r)$ to $u(r)$. This is called deformation and $L(v)-L(u_0)$ and $A(v)-f(r)$ are called homotopic in topology. If the embedding parameter p; $(0\le p\le 1)$ is considered as a 'small parameter', applying the classical perturbation technique, we can assume that the solution of (6) can be given as a power series in p, i. e.,

$$v=v_0+pv_1+p^2v_2+\dots \tag{9}$$

and setting $p=1$ results in the approximate solution of (3) as

$$u=\lim_{p\to 1} v=v_0+v_1+v_2+\dots. \tag{10}$$

3. Application of HPM

3.1. Application of HPM to the (2+1)-Dimensional Boussinesq Equation

In order to solve (1) by HPM, we choose the initial approximation

$$u(x,0,t)=f_1(x,t), \quad u_y(x,0,t)=f_2(x,t) \tag{11}$$

and construct the following homotopy:

$$u_{yy}-(u_0)_{yy}=p\left[u_{tt}-u_{xx}-(u^2)_{xx}-u_{xxxx}-(u_0)_{yy}\right]. \tag{12}$$

Assume the solution of (12) in the form

$$u(x,y,t)=u_0(x,y,t)+pu_1(x,y,t) + p^2u_2(x,y,t)+p^3u_3(x,y,t)+\dots. \tag{13}$$

Substituting (13) into (12) and collecting terms of the same power of p gives:

$$p^0: (u_0)_{yy}-(u_0)_{yy}=0, \tag{14}$$

$$p^1: (u_1)_{yy}=(u_0)_{tt}-(u_0)_{xx}-({u_0}^2)_{xx} -(u_0)_{xxxx}, \tag{15}$$

$$p^2: (u_2)_{yy}=(u_1)_{tt}-(u_1)_{xx}-(2u_0u_1)_{xx} -(u_1)_{xxxx}, \tag{16}$$

$$p^3: (u_3)_{yy}=(u_2)_{tt}-(u_2)_{xx} -(2u_0u_2+{u_1}^2)_{xx}-(u_2)_{xxxx}, \tag{17}$$

$$p^4: (u_4)_{yy}=(u_3)_{tt}-(u_3)_{xx} -(2u_0u_3+2u_1u_2)_{xx}-(u_3)_{xxxx}, \tag{18}$$

$$p^5: (u_5)_{yy}=(u_4)_{tt}-(u_4)_{xx}-\big(2u_0u_4 +2u_1u_3+{u_2}^2\big)_{xx}-(u_4)_{xxxx}, \tag{19}$$

$\dots$

We can start with the initial approximation and all the linear equations above can be easily solved, so we get all the solutions. The solution of (12) can be obtained by setting $p = 1$ in (13):

$$u(x,y,t) = u_0(x,y,t) + u_1(x,y,t) + u_2(x,y,t) + u_3(x,y,t) + u_4(x,y,t) + \dots . \quad (20)$$

3.2. Application of HPM to the (3+1)-Dimensional KP Equation

In order to solve (2) by HPM, we choose the initial approximation

$$u(x,0,z,t) = g_1(x,z,t), \quad u_y(x,0,z,t) = g_2(x,z,t)$$

and construct the following homotopy:

$$u_{yy} - (u_0)_{yy} = p(u_{xt} - 6(u^2)_x - 6uu_{xx} - u_{zz} - (u_0)_{yy}). \quad (21)$$

Assume the solution of (21) in the form

$$u(x,y,t) = u_0(x,y,t) + pu_1(x,y,t) + p^2u_2(x,y,t) + p^3u_3(x,y,t) + \dots . \quad (22)$$

Substituting (22) into (21) and collecting terms of the same power of p gives:

$$p^0 : (u_0)_{yy} - (u_0)_{yy} = 0, \quad (23)$$

$$p^1 : (u_1)_{yy} = (u_0)_{xt} - ({u_0}^2)_x - (u_0)_{xx}(u_0) - (u_0)_{xxxx} - (u_0)_{zz}, \quad (24)$$

$$p^2 : (u_2)_{yy} = (u_1)_{xt} - 2(u_0)_x(u_1)_x - (u_0)_{xx}(u_1) - (u_1)_{xx}(u_0) - (u_1)_{xxxx} - (u_1)_{zz}, \quad (25)$$

$$p^3 : (u_3)_{yy} = (u_2)_{xt} - 2(u_0)_x(u_2)_x - ({u_1}^2)_x - (u_0)_{xx}(u_2) - (u_1)_{xx}(u_1) - (u_2)_{xx}(u_0) - (u_2)_{xxxx} - (u_2)_{zz}, \quad (26)$$

$$p^4 : (u_4)_{yy} = (u_3)_{xt} - 2(u_0)_x(u_3)_x - 2(u_1)_x(u_2)_x - (u_0)_{xx}(u_3) - (u_1)_{xx}(u_2) - (u_2)_{xx}(u_1) - (u_3)_{xx}(u_0) - (u_3)_{xxxx} - (u_3)_{zz}, \quad (27)$$

$$p^5 : (u_5)_{yy} = (u_4)_{xt} - 2(u_0)_x(u_4)_x - 2(u_1)_x(u_3)_x - ({u_2}^2)_x - (u_0)_{xx}(u_4) - (u_1)_{xx}(u_3) - (u_2)_{xx}(u_2) - (u_3)_{xx}(u_1) - (u_4)_{xx}(u_0) - (u_4)_{xxxx} - (u_4)_{zz}, \quad (28)$$

...

We can start with the initial approximation and all the linear equations above can be easily solved, so we get all the solutions. The solution of (21) can be obtained by setting $p = 1$ in (22):

$$u(x,y,z,t) = u_0(x,y,z,t) + u_1(x,y,z,t) + u_2(x,y,z,t) + u_3(x,y,z,t) + u_4(x,y,z,t) + \dots . \quad (29)$$

4. Test Examples

In this section we will be concerned with the solitary wave solutions of the Boussinesq equation (1) and the KP equation (2).

In the first example, we consider the Boussinesq equation (1) which has the solitary wave solution. The solution of (1) is subject to the initial conditions

$$u(x,0,t) = K_1 - 6\alpha^2R^2\tanh^2(R(\alpha x - ct)),$$
$$u_y(x,0,t) = -12\alpha^2\beta R^3\operatorname{sech}^2(R(\alpha x - ct)) \cdot \tanh(R(\alpha x - ct)). \quad (30)$$

Using the homotopy perturbation procedure (11)–(20), we obtain following components:

$$u_0 = 0,$$
$$u_1 = K_1 - 12\alpha^2\beta R^3 y\operatorname{sech}^2(R\eta)\tanh(R\eta) - 6\alpha^2R^2\tanh^2(R\eta), \quad (31)$$

$$u_2 = \frac{3\alpha^2R^4y^2}{2}\Big[3\alpha^2 - 3c^2 - 132\alpha^4R^2 + 2\alpha^2\cosh(2R\eta) - 2c^2\cosh(2R\eta) + 104\alpha^4R^2\cosh(2R\eta) - \alpha^2\cosh(4R\eta) + c^2\cosh(4R\eta) - 4\alpha^4R^2\cosh(4R\eta)\Big]\operatorname{sech}(R\eta)^6 + \alpha^2\beta R^5y^3\Big[-9\alpha^2 + 9c^2 + 492\alpha^4R^2 - 8\alpha^2\cosh(2R\eta) + 8c^2\cosh(2R\eta) - 224\alpha^4R^2\cosh(2R\eta) + \alpha^2\cosh(4R\eta) - c^2\cosh(4R\eta) + 4\alpha^4R^2\cosh(4R\eta)\Big] \cdot \operatorname{sech}^6(R\eta)\tanh(R\eta), \quad (32)$$

$$u_3 = 3\alpha^2R^4y^2\Big[3K_1 + 54\alpha^2R^2 + 2K_1\cosh(2R\eta) - 60\alpha^2R^2\cosh(2R\eta) - K_1\cosh(4R\eta) + 6\alpha^2R^2\cosh(4R\eta)\Big]\operatorname{sech}^6(R\eta) + \frac{\alpha^2R^6y^4}{32}\Big[95\alpha^4 - 190\alpha^2c^2 + 95c^4 - 9800\alpha^6R^2 - 1920\alpha^4\beta^2R^2 + 9800\alpha^4c^2R^2 + 1249520\alpha^8R4 + 86\alpha^4\cosh(2R\eta) - 172\alpha^2c^2\cosh(2R\eta) + 86c^4\cosh(2R\eta)$$

$$
\begin{aligned}
&-172\alpha^2c^2\cosh(2R\eta)+86c^4\cosh(2R\eta)\\
&-1232\alpha^6R^2\cosh(2R\eta)-192\alpha^4\beta^2\cosh(2R\eta)\\
&+1232\alpha^4c^2\cosh(2R\eta)-1411744\alpha^8R^4\cosh(2R\eta)\\
&-32\alpha^4\cosh(4R\eta)+64\alpha^2c^2\cosh(4R\eta)\\
&-32c^4\cosh(4R\eta)+7616\alpha^6R^2\cosh(4R\eta)\\
&+1536\alpha^4\beta^2R^2\cosh(4R\eta)-7616\alpha^4c^2R^2\cosh(4R\eta)\\
&+233728\alpha^8R^4\cosh(4R\eta)-22\alpha^4\cosh(6R\eta)\\
&+44\alpha^2c^2\cosh(6R\eta)-22c^4\cosh(6R\eta)\\
&-944\alpha^6R^2\cosh(6R\eta)-192\alpha^4\beta^2R^2\cosh(6R\eta)\\
&+944\alpha^4c^2R^2\cosh(6R\eta)-8032\alpha^8R^4\cosh(6R\eta)\\
&+\alpha^4\cosh(8R\eta)-2\alpha^2c^2\cosh(8R\eta)+c^4\cosh(8R\eta)\\
&+8\alpha^6R^2\cosh(8R\eta)-8\alpha^4c^2R^2\cosh(8R\eta)\\
&+16\alpha^8R^4\cosh(8R\eta)\Big]\operatorname{sech}^{10}(R\eta)+2\alpha^4\beta R^5y^3\Big[-9K_1\\
&-210\alpha^2R^2-8K_1\cosh(2R-ct+\alpha x)\\
&+144\alpha^2R^2\cosh(2R\eta)+K_1\cosh(4R\eta)\\
&-6\alpha^2R^2\cosh(4R\eta)\Big]\operatorname{sech}(R\eta)^6\tanh(R\eta)\\
&+\frac{\alpha^2\beta R^7y^5}{80}\Big[-512\alpha^4+1030\alpha^2c^2-515c^4+60200\alpha^2R^2\\
&-60200\alpha^4c^2R^2-7215920\alpha^8R^4-596\alpha^4\cosh(2R\eta)\\
&+1192\alpha^2c^2\cosh(2R\eta)-596c^4\cosh(2R\eta)\\
&+1192\alpha^2c^2\cosh(2R\eta)-596c^4\cosh(2R\eta)\\
&+29792\alpha^6R^2\cosh(2R\eta)-29792\alpha^4c^2R^2\cosh(2R\eta)\\
&+6533824\alpha^8R^4\cosh(2R\eta)-28\alpha^4\cosh(4R\eta)\\
&+56\alpha^2c^2\cosh(4R\eta)-28c^4\cosh(4R\eta)\\
&-28448\alpha^6R^2\cosh(4R\eta)+28448\alpha^4c^2R^2\cosh(4R\eta)\\
&-749248\alpha^8R^4+(52\alpha^4-104\alpha^2c^2+52c^4+1952\alpha^6R^2\\
&-1952\alpha^4c^2R^2+16192\alpha^8R^4)\cosh(6R\eta)\\
&-(\alpha^4-2\alpha^2c^2+c^4+8\alpha^6R^2-8\alpha^4c^2R^2\\
&+16\alpha^8R^4)\cosh(8R\eta)\Big]\operatorname{sech}^{10}(R\eta)\tanh(R\eta). \qquad (33)
\end{aligned}
$$

The series solution is

$$\psi(x,t)=\sum_{m=0}^{n}u_m(x,t), \qquad (34)$$

for $n=3$, the HPM truncated series solution therefore

$$
\begin{aligned}
\psi(x,y,t)&=u_0(x,y,t)+u_1(x,y,t)\\
&\quad+u_2(x,y,t)+u_3(x,y,t)
\end{aligned} \qquad (35)
$$

and so on, where $\eta=(\alpha x-ct)$. In this manner the rest of the components of the homotopy perturbation series were obtained. Following this procedure as in the first example, substituting (31)–(33) into (35), we obtained the closed form of the soliton solution $u(x,t)$ in a close form solution

(a)

(b)

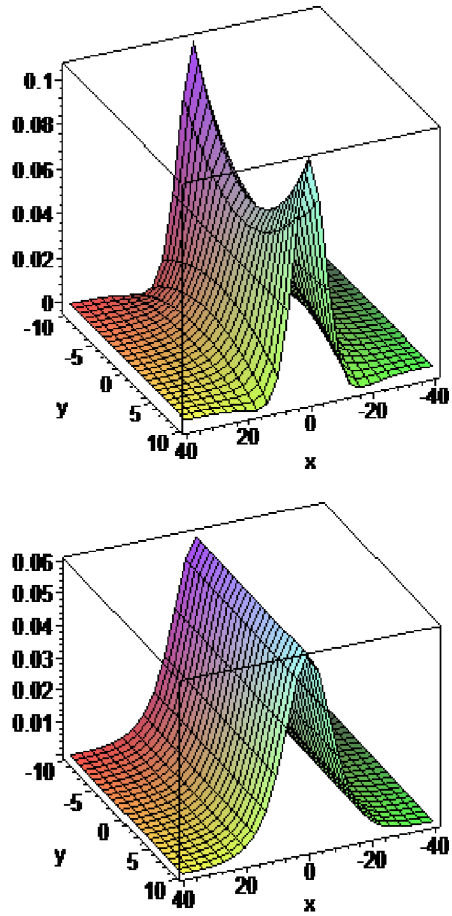

Fig. 1. (a) Truncated HPM series solution (35) and (b) for the solitary wave solution (36) of (1) with $t=0.5$ when $\alpha=5$, $\beta=0.01$, $\gamma=1$, $R=0.02$.

$$u(x,y,t)=K_1-6\alpha^2R^2\tanh^2(R(\alpha x+\beta y-ct)), \qquad (36)$$

where $c_1=\sqrt{\alpha^2+\beta^2+4\alpha^2R^2}$, $K_1=6\alpha^2R^2$, and α, β, R are arbitrary constants.

In the second example, we will consider the KP equation (2) with the initial conditions

$$
\begin{aligned}
&u(x,0,z,t)=K+2\alpha^2R^2\tanh^2(R\zeta),\\
&u_y(x,0,z,t)=4\alpha^2\beta R^3\operatorname{sech}^2(R\zeta)\tanh(R\zeta),
\end{aligned} \qquad (37)
$$

where $\zeta=(\alpha x+\beta y-ct)$, $c=-(\beta^2+\gamma^2+4\alpha^4R^2)/\alpha$, $K=-2\alpha^2R(2+R)/3$, and α, β, γ, R are arbitrary constants.

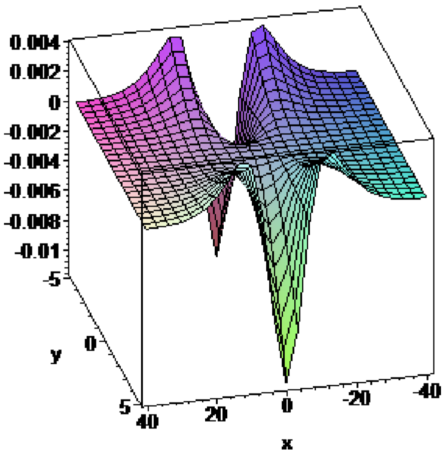

Fig. 2. Error between the solitary wave solution $u(x,t)$ and the truncated series solution $\psi(x,t)$ at $t=0.5$ when $\alpha=5$, $\beta=0.01$, $\gamma=1$, $R=0.02$.

Using the homotopy perturbation procedure (21)–(29), we obtain following components:

$$\begin{aligned} u_0 &= 0,\\ u_1 &= K+4\alpha^2\beta R^3 y\,\mathrm{sech}(R\zeta)^2\tanh(R\zeta)\\ &\quad +2\alpha^2R^2\tanh(R\zeta)^2, \end{aligned} \tag{38}$$

$$\begin{aligned} u_2 = {} & \frac{\alpha^2R^4y^2}{2}\Big[3\alpha c-3\gamma^2+132\alpha^4R^2-2\alpha c\cosh(2R\zeta)\\ & -2\gamma^2\cosh(2R\zeta)-104\alpha^4R^2\cosh(2R\zeta)+\alpha c\cosh(4R\zeta)\\ & +\gamma^2\cosh(4R\zeta)+4\alpha^4R^2\cosh(4R\zeta)\Big]\,\mathrm{sech}^6(R\zeta)\\ & +\frac{\alpha^2\beta R^5y^3}{3}\Big[9\alpha c+9\gamma^2-492\alpha^4R^2+8\alpha c\cosh(2R\zeta)\\ & +8\gamma^2\cosh(2R\zeta)+224\alpha^4R^2\cosh(2R\zeta)\\ & -\alpha c\cosh(4R\zeta)-\gamma^2\cosh(4R\zeta)\\ & -4\alpha^4R^2\cosh(4R\zeta)\Big]\,\mathrm{sech}^6(R\zeta)\tanh(R\zeta), \end{aligned} \tag{39}$$

$$\begin{aligned} u_3 = {} & \frac{\alpha^2R^6y^4}{96}\Big[-95\alpha^2c^2-190\alpha c\gamma^2-95\gamma^4\\ & +9800\alpha^5cR^2+9800\alpha^4\gamma^2R^2-1249520\alpha^8R^4\\ & -86\alpha^2c^2\cosh(2R\zeta)-172\alpha c\gamma^2\cosh(2R\zeta)\\ & -86\gamma^4\cosh(2R\zeta)+1232\alpha^5cR^2\cosh(2R\zeta)\\ & +1232\alpha^4\gamma^2R^2\cosh(2R\zeta)+1411744\alpha^8R^4\cosh(2R\zeta)\\ & +32\alpha^2c^2\cosh(4R\zeta)+64\alpha c\gamma^2\cosh(4R\zeta)\\ & +32\gamma^4\cosh(4R\zeta)-7616\alpha^5cR^2\cosh(4R\zeta)\\ & -7616\alpha^4\gamma^2R^2\cosh(4R\zeta)-233728\alpha^8R^4\cosh(4R\zeta)\\ & +22\alpha^2c^2\cosh(6R\zeta)+44\alpha c\gamma^2\cosh(6R\zeta)\\ & +22\gamma^4\cosh(6R\zeta)+944\alpha^5cR^2\cosh(6R\zeta)\\ & +944\alpha^4\gamma^2R^2\cosh(6R\zeta)+8032\alpha^8R^4\cosh(6R\zeta)\\ & -\alpha^2c^2\cosh(8R\zeta)-2\alpha c\gamma^2\cosh(8R\zeta)-\gamma^4\cosh(8R\zeta)\\ & -8\alpha^5cR^2\cosh(8R\zeta)-8\alpha^4\gamma^2R^2\cosh(8R\zeta)\\ & -16\alpha^8R^4\cosh(8R\zeta)\Big]\,\mathrm{sech}^{10}(R\zeta)\\ & +\frac{\alpha c\beta R^7y^5}{240}\Big[512\alpha^2c^2+1030\alpha c\gamma^2+515\gamma^4\\ & -60200\alpha^5cR^2-60200\alpha^4\gamma^2R^2+7215920\alpha^8R^4\\ & +596\alpha^2c^2\cosh(2R\zeta)+1192\alpha c\gamma^2\cosh(2R\zeta)\\ & +596\gamma^4\cosh(2R\zeta)-29792\alpha^5cR^2\cosh(2R\zeta)\\ & -29792\alpha^4\gamma^2R^2\cosh(2R\zeta)-6533824\alpha^8R^4\cosh(2R\zeta)\\ & +28\alpha^2c^2\cosh(4R\zeta)+56\alpha c\gamma^2\cosh(4R\zeta)\\ & +28\gamma^4\cosh(4R\zeta)+28448\alpha^5cR^2\cosh(4R\zeta)\\ & +28448\alpha^4\gamma^2R^2\cosh(4R\zeta)+749248\alpha^8R^4\cosh(4R\zeta)\\ & -52\alpha^2c^2\cosh(6R\zeta)-104\alpha c\gamma^2\cosh(6R\zeta)\\ & -52\gamma^4\cosh(6R\zeta)-1952\alpha^5cR^2\cosh(6R\zeta)\\ & -1952\alpha^4\gamma^2R^2\cosh(6R\zeta)-16192\alpha^8R^4\cosh(6R\zeta)\\ & +\alpha^2c^2\cosh(8R\zeta)+2\alpha c\gamma^2\cosh(8R\zeta)+\gamma^4\cosh(8R\zeta)\\ & +8\alpha^5cR^2\cosh(8R\zeta)+8\alpha^4\gamma^2R^2\cosh(8R\zeta)\\ & +16\alpha^8R^4\cosh(8R\zeta)\Big]\,\mathrm{sech}^{10}(R\zeta)\tanh(R\zeta). \end{aligned} \tag{40}$$

With the series solution

$$\psi(x,t)=\sum_{m=0}^{n}u_m(x,t), \tag{41}$$

for $n=3$, we obtain the HPM truncated series solution as

$$\begin{aligned} \psi(x,y,z,t) &= u_0(x,y,z,t)+u_1(x,y,z,t)\\ &\quad +u_2(x,y,z,t)+u_3(x,y,z,t), \end{aligned} \tag{42}$$

where $\zeta=(\alpha x+\gamma y-ct)$, $c=-(\beta^2+\gamma^2+4\alpha^4R^2)/\alpha$, $K=-2\alpha^2R(2+R)/3$, and α, β, γ, R are arbitrary constants. In this manner the rest of the components of the decomposition series were obtained. Substituting $u_0=0$ and (38)–(40) into (42) gives the solution $u(x,y,z,t)$ in a series form and the series can be written in a closed form solution by

$$\begin{aligned} u(x,y,z,t) &= K+2\alpha^2R^2\tanh^2(R(-(ct)\\ &\quad +\alpha x+\beta y+\gamma z)), \end{aligned} \tag{43}$$

where $c=-(\beta^2+\gamma^2+4\alpha^4R^2)/\alpha$, $K=-2\alpha^2R(2+R)/3$, and α, β, γ, R are arbitrary constants. This result can be verified through substitution [6].

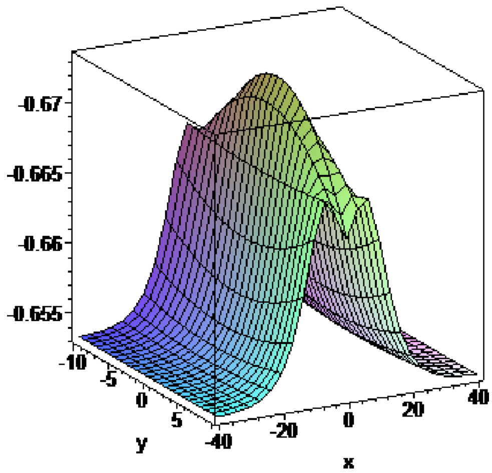

Fig. 3. Truncated HPM series solution (42) for (2) with $t = z = 0.5$ when $\alpha = 5$, $\gamma = 1$, $R = 0.02$.

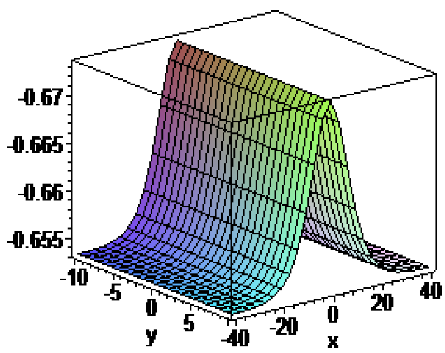

Fig. 4. Solitary wave solution (43) of (2) at $t = z = 0.5$ when $\alpha = 5$, $\beta = 0.01$, $\gamma = 1$, $R = 0.02$.

In the third example, we will consider the KP equation (2) with the initial conditions for numerical comparison purpose as

$$\begin{aligned} u(x,0,z,t) &= K + 2\alpha^2 R^2 \tanh(R\zeta), \\ u_y(x,0,z,t) &= 4\alpha^2 \beta R^3 \operatorname{sech}^2(R\zeta)\tanh(R\zeta), \end{aligned} \quad (44)$$

where $\zeta = (\alpha x + \gamma y - ct)$, $c = -(\beta^2 + \gamma^2 + 4\alpha^4 R^2)/\alpha$, $K = -2\alpha^2 R(2+R)/3$, and α, β, γ, R are arbitrary constants.

Using homotopy perturbation procedure (21)–(29), we obtain following components:

$$\begin{aligned} u_0 &= 0, \\ u_1 &= \int_0^y \int_0^y \big[(u_0)_{xt} - ({u_0}^2)_x - (u_0)_{xx}(u_0) \\ &\qquad - (u_0)_{xxxx} - (u_0)_{zz}\big]\,\mathrm{d}y\mathrm{d}y, \end{aligned}$$

$$\begin{aligned} u_2 &= \int_0^y \int_0^y \big[(u_1)_{xt} - 2(u_0)_x(u_1)_x - (u_0)_{xx}(u_1) \\ &\qquad - (u_1)_{xx}(u_0) - (u_1)_{xxxx} - (u_1)_{zz}\big]\,\mathrm{d}y\mathrm{d}y, \\ &\cdots \end{aligned} \quad (45)$$

and the exact solution

$$\begin{aligned} u(x,y,z,t) = K + 2\alpha^2 R^2 \tanh^2(R(-(ct) \\ + \alpha x + \beta y + \gamma z))^2, \end{aligned} \quad (46)$$

where $\zeta = (\alpha x + \gamma y - ct)$, $c = -(\beta^2 + \gamma^2 + 4\alpha^4 R^2)/\alpha$, $K = -2\alpha^2 R(2+R)/3$, and α, β, γ, R are arbitrary constants.

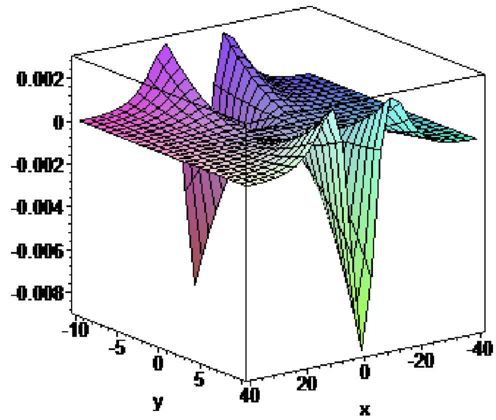

Fig. 5. Error between the solitary wave solution $u(x,t)$ (43) and the truncated series solution $\psi(x,t)$ at $t = z = 0.5$ when $\alpha = 5$, $\beta = 0.01$, $\gamma = 1$, $R = 0.02$.

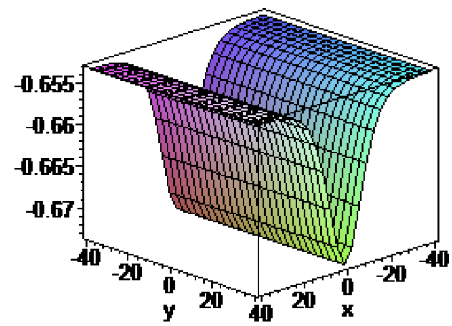

Fig. 6. Solitary wave solution (46) of (2) at $t = z = 0.5$ when $\alpha = 5$, $\beta = 0.01$, $\gamma = 1$, $R = 0.02$.

5. Conclusion

In this study, the HPM was used for solving the Boussinesq equation (1) and the KP equation (2) with initial conditions. We compared the approximation solution with the exact solution of the corresponding equation. Numerical approximations show a high degree of accuracy. The numerical results we obtained justify the advantage of this methodology, even in the few terms the approximation is accurate.

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